STATISTICS

Paper – II

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) Derive a control chart to test $\mu = 400$ given that $\sigma = 30$ such that the chances of a point going out of the control limits when the process really is in control are 1 in 20 while the chances of detecting a shift of μ to 425 within two samples after a shift occurs are 0·99 percent. For the same chart, what are the chances of detecting a shift of μ from 400 to 380 within two samples ? *8*

(b) Define renewal process. Derive the renewal function. *8*

(c) Represent a general linear programming problem consisting of n variables and m constraints, in its canonical and standard forms. Given the following linear programming problem :

Maximize $z = x_1 + 1{\cdot}5x_2$

subject to the constraints

$$2x_1 + 2x_2 \leq 160$$

$$x_1 + 2x_2 \leq 120$$

$$4x_1 + 2x_2 \leq 280$$

$$x_1, x_2 \geq 0$$

find the solution of the problem using graphical method. *8*

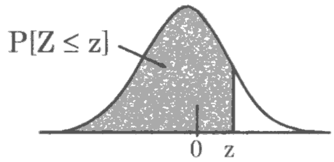
(d) Define the pay-off matrix in a two-person zero-sum rectangular game. Obtain a necessary and sufficient condition under which the game possesses a saddle point. *8*

(e) A study about a population showed that the mobility of the population of a state to a village, town and city is in the following percentages :

From	To		
	Village	Town	City
Village	60	25	15
Town	10	70	20
City	10	30	60

What will be the proportion of population in the village, town and city after one year and two years, given that the present population has proportions of 0·50, 0·40 and 0·10 in the village, town and city respectively ? *8*

Standard Normal Probabilities



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

Q2. (a) Find a sequential plan for which, in the usual notation $p_1 = 0{\cdot}01$, $\alpha = 0{\cdot}05$, $p_2 = 0{\cdot}06$ and $\beta = 0{\cdot}10$. Display the plan in a table for the first 46 units. *15*

(b) (i) A system is composed of five identical independent elements in parallel. What should be the reliability of each element to achieve a system reliability of 0·96 ? *7*

(ii) Three identical units each with a reliability of 0·9 for a system are operating in parallel. What is the system reliability if at least one of the units is required for the system to be successful ? If you add another unit with identical reliability characteristics, what will be the increase in system reliability ? *8*

(c) Consider a system where $m - 1$ units are standing by to support one basic unit. Obtain the system reliability assuming the units are identical. Show that for a two unit system the reliability of a standby system is better than that of an active redundant system. *10*

Q3. (a) What is the basic philosophy behind sensitivity analysis ? What are the issues which could be answered using sensitivity analysis in linear programming problems ?

For the following linear programming problem :

Maximize $z = 3x_1 + 2x_2$

subject to the constraints

$$x_1 + 2x_2 \le 6$$

$$2x_1 + x_2 \le 8$$

$$-x_1 + x_2 \le 1$$

$$x_2 \le 2$$

$$x_1, x_2 \ge 0$$

the optimal simplex table is obtained as

i	Basis	Cost	Solution	3	2	0	0	0	0
				x_1	x_2	s_1	s_2	s_3	s_4
1	x_2	2	4/3	0	1	2/3	−1/3	0	0
2	x_1	3	10/3	1	0	−1/3	2/3	0	0
3	s_3	0	3	0	0	− 1	1	1	0
4	s_4	0	2/3	0	0	−2/3	1/3	0	1
		z	38/3	0	0	1/3	4/3	0	0

where s_i (i = 1, 2, 3, 4) denotes the slack variable used in the i^{th} constraint.

(i) Find the status and worth of each resource.

(ii) Find the possible range of the cost coefficient associated with the variable x_1 for which the optimum solution remains unchanged. *15*

(b) Explain what is a Transportation Problem (TP). Differentiate between balanced and unbalanced transportation problems.

Four petrol dealers A, B, C, D require 50,000, 40,000, 60,000 and 40,000 litres of petrol respectively. It is possible to supply these demands from three locations 1, 2 and 3 which have 80,000, 1,00,000 and 50,000 litres respectively. The costs (in rupees per thousand litres) to ship the petrol from different locations to different dealers are shown in the following table :

Location	Dealer				Supply
	A	B	C	D	
1	70	60	60	60	8
2	50	80	60	70	10
3	80	50	80	60	5
Required	5	4	6	4	

(The supply and requirement values in the table have been multiplied by 10^{-4} to make computations easier)

Determine the amount of petrol to be shipped from each location to each dealer so that all the dealers' requirements are satisfied and the total shipping costs are a minimum. *15*

(c) Prove that if the pair of linear programming problems

Maximize $Z = C'X$
subject to

$$AX \leq b$$
$$X \geq 0$$

and

Minimize $Z^* = b'Y$
subject to

$$A'Y \geq C$$
$$Y \geq 0$$

have feasible vectors X and Y, then they have optimal vectors such that Maximum $C'X$ = Minimum $b'Y$. *10*

Q4. (a) Construct a V-mask for the following data with $\mu = 10$ and $\sigma = 1$ and comment. (Given $\alpha = 0{\cdot}0027$, $\beta = 0{\cdot}01$, $\delta = 1$) *15*

Period i	x_i	Period i	x_i	Period i	x_i
1	9·45	11	9·03	21	10·90
2	7·99	12	11·47	22	9·33
3	9·29	13	10·51	23	12·29
4	11·66	14	9·40	24	11·50
5	12·16	15	10·08	25	10·60
6	10·18	16	9·37	26	11·08
7	8·04	17	10·62	27	10·38
8	11·46	18	10·31	28	11·62
9	9·20	19	8·52	29	11·31
10	10·34	20	10·84	30	10·52

(b) Distinguish between deterministic and probabilistic inventory problems. Discuss the continuous case of a probabilistic inventory model with instantaneous demand and no set-up cost.

A sweet company sells chocolate-based sweets by the kilogram (kg). On every kg of these sweets sold on the day of its preparation, the company makes a profit of ₹ 0·55 per kg. The company disposes of all the chocolate-based sweets not sold on the date of its preparation, at a loss of ₹ 0·22 per kg. If the demand is known to be rectangular between 2000 and 4000 kg, determine the optimum daily amount of sweets prepared. *10*

(c) Obtain the steady-state solution for P_n, the probability of n customers at any arbitrary point of time in the (M/M/C) : (∞/FIFO) queue system. Hence, find the formulae for (i) Traffic intensity, (ii) The probability that an arriving customer has to wait, and (iii) The average number of customers in the queue. *15*

SECTION B

Q5. (a) What is an index number ? Describe the general lines on which you proceed to construct a cost of living index for factory workers in an industrial area. *8*

(b) Explain seemingly unrelated regression model. Obtain the generalized least squares estimator of the parameter vector when the covariance matrix is diagonal. *8*

(c) Explain the two series of acreage statistics available in India. *8*

(d) What are the defects in comparing mortality situations between two regions by means of their crude death rates ? Explain how these defects can be eliminated. *8*

(e) Describe different methods of converting raw scores into some equivalent comparable scores in Psychometry. *8*

Q6. (a) (i) Obtain the autocorrelation function of AR(2) process and show that the process is covariance stationary. *8*

(ii) Obtain the best forecast in the mean square sense for ARIMA (p, d, q) process. *7*

(b) For a model :

$$y_{1t} = \beta_{12}\, y_{2t} + \gamma_{11}\, x_{1t} + u_{1t}$$

$$y_{2t} = \beta_{21}\, y_{1t} + \gamma_{22}\, x_{2t} + \gamma_{23}\, x_{3t} + u_{2t}$$

$$X'X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{pmatrix} \text{ and } X'Y = \begin{pmatrix} 5 & 10 \\ 40 & 20 \\ 20 & 30 \end{pmatrix}$$

Determine the identifiability of each equation and estimate the parameters by using an appropriate method. *15*

(c) Explain the existing machinery for collection of births and deaths in India and its shortcomings. *10*

Q7. (a) Describe the major sources of demographic data in India. *15*

(b) Calculate (i) General Fertility Rate (ii) Total Fertility Rate, and (iii) Gross Reproduction Rate from the following data.

Age group of females	No. of females ('000)	Total Births
15 – 19	16·0	260
20 – 24	16·4	2244
25 – 29	15·8	1894
30 – 34	15·2	1320
35 – 39	14·8	916
40 – 44	15·0	280
45 – 49	14·5	145

Assume that the proportion of female births is 46·2 percent. *15*

(c) Explain the concept of validity of a test. Discuss the different types of validity. *10*

Q8. (a) Compute Laspeyres' quantity and price index numbers for 2005.

Commodity	1995		2005	
	Quantity	Value	Quantity	Value
A	50	350	60	420
B	120	600	140	700
C	30	330	20	200
D	20	360	15	300
E	5	40	5	50

Also show that Fisher's ideal index number satisfies factor reversal test. *15*

(b) The model appropriate for the following data is given by $\bar{Y}_i = \alpha + \beta \bar{X}_i + \bar{u}_i$ where $v(\bar{u}_i) = \frac{\sigma^2}{n_i}$, i = 1, 2, ... m. Obtain the generalized least square estimates and their variances. *15*

n_i :	12	6	11	10	11
$\bar{X}_i$:	2	3	4	5	6
$\bar{Y}_i$:	4	7	3	9	17

(c) A part of a life table is given with most of the entries missing. On the basis of the available figures, complete the missing ones. *10*

Age x	l_x	d_x	q_x	L_x	T_x	e_x^0
87	34	–	0·2353	–	–	–
88	–	–	0·2308	–	–	–
89	–	–	0·2500	–	–	–
90	–	–	0·2667	–	35·5	–