

STATISTICS

Paper – II

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) Distinguish between defects and defectives in SQC. Give some examples of defects for which the c-chart is applicable. How do you calculate control limits for a c-chart ? 8

(b) If T is a random variable representing the hours of failure for a device with following pdf :

$$f(t) = t e^{-t^2/2}; t \geq 0$$

find the reliability function and hazard function. If 50 devices are placed in operation and 27 are still in operation 1 hour later, find approximately the expected number of failures in the time interval from 1 to 1.1 hours using hazard function. 8

(c) Solve the following linear programming problem graphically : 8

$$\text{Minimize } Z = 20x_1 + 20x_2$$

subject to

$$x_1 + 2x_2 \geq 1$$

$$2x_1 + x_2 \geq 2$$

$$2x_1 + 3x_2 \geq 3$$

$$3x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

(d) A company has a team of four salesmen and there are four districts where the company wants to start its business. The profit per day (in ₹) for each salesman in each district is estimated as given below :

		Districts			
		1	2	3	4
Salesman	A	16	10	14	11
	B	14	11	15	15
	C	15	15	13	12
	D	13	12	14	15

Find the assignment of salesmen to various districts which will yield maximum profit. 8

- (e) Using the Rule of Dominance, solve the game with the following pay-off matrix :

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		Player B's strategies				
		1	2	3	4	5
Player A's strategies	1	2	5	10	7	2
	2	3	3	6	6	4
	3	4	4	8	12	1

- Q2.** (a) Show the probability that at least one of the two points \bar{X} and R goes outside the control limits

$$1 - \left[S(\sqrt{n} T - 3\rho) - S(\sqrt{n} T + 3\rho) \right] \cdot \left[P\left(\frac{R}{\sigma} \leq D_2 \rho\right) - P\left(\frac{R}{\sigma} \leq D_1 \rho\right) \right]$$

where $\rho = \frac{\sigma'}{\sigma}$ $T = \frac{\mu' - \mu}{\sigma}$

$$S(x) = \int_x^{\infty} e^{-\frac{1}{2}t^2} dt$$

assuming that the control charts are based on μ' population mean and σ' as population standard deviation, where the actual values of these parameters are μ and σ respectively.

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- (b) The time to failure in operating hours of a critical solid state power unit has the hazard rate function given by

$$\lambda(t) = (0.003) \left(\frac{t}{500} \right)^{1/2} ; \text{ for } t \geq 0$$

- (i) What is the reliability if the power unit must operate continuously for 50 hours ?
- (ii) Determine the design life if a reliability of 0.90 is desired.
- (iii) Compute the MTTF.
- (iv) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation ?

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(c) Use Simplex method to find an optimal solution to the following LPP :

$$\text{Maximize } Z = 3x_1 + 2x_2$$

subject to

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Does it indicate the existence of an alternative optimal solution ? If yes, obtain it.

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Q3. (a) Define renewal function and renewal density function. Derive the following forward renewal equation :

$$U(t) = f_1(t) + \int_0^t U(t - \tau) f(\tau) d\tau$$

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(b) The Optimal Simplex table of the following LPP :

$$\text{Maximize } Z = 10x_1 + 15x_2 + 20x_3$$

subject to

$$2x_1 + 4x_2 + 6x_3 \leq 24$$

$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

is given below :

C_B	X_B	C_j					b
		10	15	20	0	0	
		x_1	x_2	x_3	x_4	x_5	
20	x_3	0	-1	1	1/2	-1/3	2
10	x_1	1	5	0	-1	1	6
	$Z_j - C_j$	0	15	0	0	10/3	Z=100

Perform the sensitivity analysis to check whether optimality of the table is affected if the profit coefficients are changed from (10, 15, 20) to (7, 14, 15).

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(c) Show that the Markov Chain with transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

is irreducible, periodic and all its states are persistent.

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Q4. (a) The number of customer complaints received daily by an organisation is given below :

Day	Complaint
1	2
2	3
3	0
4	1
5	9
6	2
7	0
8	0
9	4
10	2
11	0
12	7
13	0
14	2
15	4

Does it mean that the number of complaints is under statistical control ?

Establish a control scheme for the future. Draw control chart.

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- (b) At a certain petrol pump, customers arrive in a Poisson process with an average time of 5 minutes between arrivals. The time intervals between services at the petrol pump follow exponential and as such the mean time taken to service a unit is 2 minutes. Then find the average number of customers in the queuing system and how long, on an average, a customer has to wait in the queue ? 10

- (c) Write the applications of duality. Use Dual Simplex method to find the solution of the following LPP : 15

Minimize $Z = x_1 + 4x_2 + 3x_4$

subject to

$$x_1 + 2x_2 - x_3 + x_4 \geq 3$$

$$-2x_1 - x_2 + 4x_3 + x_4 \geq 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

SECTION B

Q5. (a) Explain seasonal component of a time series. Explain Ratio to Moving Average method for measurement of seasonal variation in a time series data. 8

(b) For the General Linear Model given by

$$\mathbf{Y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{U}$$

with assumptions $E(\mathbf{U}) = 0$, $E(\mathbf{U}\mathbf{U}') = \sigma^2\mathbf{I}_n$, \mathbf{X} is a set of fixed numbers with $\rho(\mathbf{X}) = k < n$, $\mathbf{B} = (\beta_1, \beta_2, \dots, \beta_k)'$ and $\mathbf{U} = (U_1, U_2, \dots, U_n)'$.

Obtain ordinary least square estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ and show that it is unbiased estimator of $\boldsymbol{\beta}$. Hence find $\text{Var}(\hat{\boldsymbol{\beta}})$. 8

(c) Define Age-specific Fertility Rate and the Total Fertility Rate (TFR). Also define Gross Reproduction Rate (GRR). 8

(d) Describe the logistic growth model in population projection. State the situation when time series data would follow logistic law. Also state the limitation. 8

(e) What is measurement scale in statistics ? Also describe the types of measurement scales with example in each type. 8

Q6. (a) Define Autocorrelation of order k and Correlogram. For an infinite series generated by the moving average of a random component with equal weights, the correlogram is given by

$$r_k = \begin{cases} 1 - \frac{k}{m} & \text{for } k \leq m \\ 0 & \text{for } k > m \end{cases} . \quad 15$$

- (b) State the need of standardization of Crude Death Rate (CDR). Also compute and compare the standardized CDR of the populations A and B in the following table :

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Age group (in years)	Death rate per 1000 population		Standard Population
	Population A	Population B	
0 – 4	3.0	10.0	1000
5 – 14	2.8	6.0	1500
15 – 24	1.0	2.0	1100
25 – 34	0.8	2.0	900
35 – 44	2.0	3.0	800
45 – 59	4.0	6.5	500
60 – 74	10.0	15.0	400
≥ 75	25.0	30.0	300

- (c) (i) Describe standardization of scale by Z-score and Min-Max scaling in psychometric measurements.
- (ii) Describe percentile and find the percentile from the following data :

The scores obtained by 10 students are 38, 47, 49, 58, 60, 65, 70, 79, 80, 92. Using the percentile formula, calculate the percentile for score 70.

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- Q7.** (a) Calculate Fisher's Ideal Index from the following data and verify that it satisfies time and factor reversal tests :

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Commodity	2015 – 16		2016 – 17	
	Price	Value	Price	Value
A	10	100	12	96
B	8	96	8	104
C	12	144	15	120
D	20	300	25	250
E	5	40	8	64
F	2	20	4	24

- (b) Describe various columns of a life table. Also, fill in the blanks in the portion of the table given below with question mark (?). 15

Age (X) in years	l_X	d_X	p_X	q_X	L_X	T_X	e_X^0
4	100000	500	?	?	?	5110000	?
5	?	400	?	?	?	?	?

- (c) Describe the present statistical system in India and the role of National Statistical Office. 15

- Q8.** (a) Prove the following relationship between the force of mortality (μ_X) at age X and the expectation of life (e_X^0) at age X.

$$\mu_X = \frac{1}{e_X^0} \left[1 + \frac{d}{dX} e_X^0 \right]. \quad 15$$

- (b) What are the consequences of multicollinearity and the consequences of auto correlated disturbances? For the General linear model given by

$$\mathbf{Y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{U}, \text{ where } \boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_k)', \mathbf{U} = (U_1, U_2, \dots, U_n)'$$

with the assumption $E(\mathbf{U}) = 0$ and $E(\mathbf{U}\mathbf{U}') = \sigma^2 \boldsymbol{\Omega}$ where σ^2 is unknown but $\boldsymbol{\Omega}$ is known symmetric positive definite matrix of order n.

Obtain the generalized least squares estimator $\hat{\mathbf{b}}$ of $\boldsymbol{\beta}$ with variance of $\hat{\mathbf{b}}$. Also obtain unbiased estimator of σ^2 . 15

- (c) What is the significance of cost of living index number? The following table gives the per capita income and the cost of living index of a community. Calculate the real income taking into account the rise in the cost of living. 10

Year	Cost of living index (Base 2001)	Per capita income (₹ in "000") per year
2001	100	360
2002	104	400
2003	115	480
2004	160	520
2005	210	550
2006	260	590
2007	300	610
2008	320	650

