JKLO–B–STSC

STATISTICS

PAPER—II

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. **1** and **5** are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

SECTION—A

1. *(a)* Develop survival function, when the hazard rate of the system is described by the function $\mu(t) = a + 2bt$; $a > 0$, $b > 0$ and $t > 0$. Also write the name of the distribution, which a life pattern of a system follows. 8

(b) The average fraction defective of a large sample of a product is 0·1537. Calculate the control limits using subgroup size as 2000. What modification do you need, if the subgroup size is not constant? 8

(c) Define the term reliability function and compute hazard rate [$h(t)$], when the life pattern of a system is described by the log-normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma x} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}; \ \mu > 0, \ \sigma > 0 \text{ and } x > 0$$

Also compute $h(e^{\mu})$. 8

(d) Why is simulation used? Explain how simulation can be applied in the case of inventory control, where the demand is probabilistic and lead time is fixed. 8

(e) Solve the game whose payoff matrix is given below :

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix} \end{array}$$

8

2. *(a)* Explain the purpose of $\bar{x}$-chart, R-chart, p-chart and the parameters that must be known or estimated to establish their control limits and also give interpretation of the above charts. 15

(b) Consider the following transition probability matrix :

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}; \quad 0 < \alpha, \ \beta < 1$$

Show that the process is ergodic. 10

(c) Explain Box-Jenkins methodology for ARIMA (p, d, q) model. 15

3. (a) Explain single sampling plan. Obtain its OC and ATI functions. Also determine the constants in the single sampling plan. 15

(b) Explain series and parallel systems and their reliability functions. Consider a system, consisting of n components, such that the failure of the i th component occurs in accordance with a Poisson process of fixed intensity a. Compute the reliability and expected life of the system, under series and parallel system structures. 15

(c) What is heteroscedasticity? What are its consequences? Describe Goldfeld-Quandt test. 10

4. (a) Define the terms renewal density and renewal function. If the renewal function is $M(t) = E(N(t))$, where $N(t)$ is the number of renewals in $[0, t]$, show that

$$M(t) = F(t) + \int_0^t M(t-x)\, dF(x)$$

Also show that for large t, the average number of renewals per unit time converges to $\frac{1}{\mu}$, where μ is mean of inter-arrival time between two consecutive renewals. 15

(b) Use two-phase simplex method to solve the following LP problem :

Minimize $z = x_1 + x_2$

subject to the constraints

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

15

(c) Let the probability function of demand of a certain item during a week be

$$f(x) = \frac{x}{50}, \quad 0 \leq x \leq 50$$
$$= 0, \quad \text{otherwise}$$

The unit carrying cost of the item in inventory is ₹ 2·00 per week and unit storage cost is ₹ 8·00 per week. Determine the optimal order of the inventory. 10

SECTION—B

5. *(a)* Explain the two-stage least squares method of estimation for the following simultaneous system :

$$C_t = \alpha + \beta Y_t + u_t$$
$$Y_t = C_t + Z_t$$

where C = aggregate consumption expenditure, Y = national income, Z = non-consumption expenditure and u_t = a stochastic disturbance term. 8

(b) Explain the model M/G/1. Obtain Pollaczek-Khinchine formula. 8

(c) What do you mean by expectation of life at a given age? Is it correct to say that a man aged 65 years is expected to live two more years when the expectation of life is 67 years? Explain. 8

(d) What is the difference between vital statistics and population statistics? Write down the various uses of vital statistics for a country. 8

(e) How is intelligence measured? Explain the terms mental age and intelligence quotient. How is the validity of an intelligence test generally determined? 8

6. *(a)* Explain the uses and limitations of index numbers. Calculate Laspeyres', Paasche's and Fisher's indices for the following data. Also examine which of the above indices satisfy *(i)* time reversal test and *(ii)* factor reversal test :

Commodity	*Base Year*		*Current Year*	
	Price	*Quantity*	*Price*	*Quantity*
P	6·5	500	10·8	560
Q	2·8	124	2·9	148
R	4·7	69	8·2	78
S	10·9	38	13·4	24
T	8·6	49	10·8	27

15

(b) Discuss the collection and publication of trade statistics that are available in India and their defects, if any. 15

(c) Write a note on inter-censal and post-censal methods for population estimates. 10

7. *(a)* Explain general linear model. Obtain the OLS estimators of the parameters. Show that the estimators are BLUEs. Define multiple correlation coefficient. Explain its use. 15

(b) Define and compare various measures of fertility. Compute gross reproduction rate and net reproduction rate from the data given below :

Age (Years)	*Fertility Rate*	*Survival Rate*
15–19	0·0108	0·969
20–24	0·0662	0·967
25–29	0·0675	0·963
30–34	0·0413	0·958
35–39	0·0216	0·952
40–44	0·0063	0·942
45–49	0·0004	0·928

15

(c) Explain the system of collection and publication of agricultural statistics in India. Describe the components of agriculture improvement division. 10

8. *(a)* Establish the autocorrelation coefficients of AR(2) process. Explain Zellner's seemingly unrelated regression model and a method of estimating the seemingly unrelated regressions. 10

(b) Distinguish between the central death rate (m_x), the probability of death (q_x) and the force of mortality (μ_x) in a life table. Indicate possible inter-relationship among these measures, stating necessary assumptions. 15

(c) Explain the various methods of scaling scores bringing out the underlying importance of normal distribution in psychometry. 15

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