DFG-D-STSC

STATISTICS
Paper – II

*Time Allowed : **Three** Hours*

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION A

Q1. (a) Explain different causes of variation in the quality of manufactured products. Also, what are the criteria for detecting lack of control in $\bar{X}$ and R charts ? *8*

(b) Explain item-by-item, single sample sequential sampling plan. Also explain lot-by-lot inspection plan for variables. *8*

(c) Explain a parallel system and its parameters. *8*

(d) Stating the primal – dual relationship in linear programming problems, write the dual of the following primal problem :

Maximize $z = 2x_1 - x_2 + x_3$

subject to

$$2x_1 + 3x_2 - 5x_3 \geq 4$$

$$-x_1 + 9x_2 - x_3 \geq 3$$

$$4x_1 + 6x_2 + 3x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0.$$ *8*

(e) Define a Markov chain. What do you understand by a Markov chain of order k ? Given the following T.P.M. of a Markov chain with states (0, 1),

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{array}$$

find the two-step transition matrix and hence find $p_{11}^{(2)}$. *8*

Q2. (a) A convenient notation for summarizing the characteristics of a queuing model is generally given by the format : (a/b/c) : (d/e). What are the characteristics of the queue which these letters show ?

Find the distribution of a single-server queuing model with finite queue size, Poisson arrival and service rates, if the services are provided with first come, first served basis. *15*

(b) Mention different control charts for attributes. Also describe the method of constructing p-chart. *15*

(c) What do you mean by the Transition Probability Matrix (T.P.M.) of a Markov chain ? Mention some of its properties.

Let $\{X_n\}$ be a Markov chain with state space {0, 1, 2}, initial probability vector $P(0) = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ and one-step transition matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix} \end{array}$$

(i) Compute $P[X_0 = 0, X_1 = 1, X_2 = 1]$.

(ii) Show that $P[X_1 = 1 \text{ and } X_2 = 1 \mid X_0 = 0] = p_{01}p_{11}$.

(iii) Compute $p_{01}^{(2)}$. *10*

Q3. (a) Differentiate between maintainability and availability. Also derive the expression for maintainability. *15*

(b) Explain double sampling plan by attributes and compute ASN, ATI and OC curve for the same plan. *15*

(c) Express a Transportation Problem (T.P.) in the form of a linear programming problem. Show that in a T.P. with m ports and n destinations, the number of basic variables will be m + n – 1. *10*

Find the optimal solution of the following T.P. :

		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Ports	O_1	2	1	3	3	2	5	50
	O_2	3	2	2	4	3	4	40
	O_3	3	5	4	2	4	1	60
	O_4	4	2	2	1	2	2	31
Demand		30	50	20	40	30	11	

Q4. (a) Define a Linear Programming Problem (LPP) with m constraints and n variables x_i (i = 1, 2, ..., n); (m < n). Show that the set of all feasible solutions to an LPP is a convex set.

Find the solution of the following LPP :

Minimize $z = 5x_1 - 4x_2 + 6x_3 + 8x_4$

subject to

$$x_1 + 7x_2 + 3x_3 + 7x_4 \leq 46$$

$$3x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$2x_1 + 3x_2 - x_3 + x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0.$$ *15*

(b) Define renewal process. For $k \in N$ and $t \geq 0$, show that $\{N_t \geq k\}$, if and only if $\{S_k \leq t\}$. *15*

(c) What is a general inventory model ? Describe the various costs involved in it. Obtain the expression of optimum order quantity in an inventory model with constant rate of demand, instantaneous order replacement and no shortage. *10*

SECTION B

Q5. (a) Define time series data and explain moving average for a multiplicative model. *8*

(b) Describe the problem of 'multicollinearity' in econometrics and explain in brief how you will detect it. *8*

(c) Derive an algebraic expression relating the probability of a person dying between the age of x and (x + 1), q_x to the force of mortality, μ_x. *8*

(d) Distinguish between stable and stationary populations. *8*

(e) What are σ and standard scores ? The fifth grade norms for a reading examination are Mean = 60 and SD = 10; for an arithmetic examination Mean = 26 and SD = 4. A student scores 55 on the reading and 24 on the arithmetic test. Compare his σ scores. In which test is he better ? *8*

Q6. (a) For the following data, construct the cost of living index for the year 2013 (Base 1990 = 100) using the method of weighted price relatives : *15*

Item	Unit	Price (1990)	Price (2013)	Weight
A	Kg.	50	75	10%
B	Litre	60	75	25%
C	Dozen	200	240	20%
D	Kg.	80	100	40%
E	One pair	160	200	5%

(b) State the problem of identification. Explain with suitable notation, the rank and order conditions for identifiability.

Identify the following system : *15*

$$y_1 = 3y_2 - 2x_1 + x_2 + U_1$$

$$y_2 = y_3 + x_3 + U_2$$

$$y_3 = y_1 - y_2 - 2x_3 + U_3$$

(c) Compute the standardised death rates for two countries A and B from the following data and compare them : *10*

Age group (years)	Death Rate per 1,000		Standardised Population (in lakhs)
	Country A	Country B	
0 – 4	20·0	5·0	100
5 – 14	1·0	0·5	200
15 – 24	1·4	1·0	190
25 – 34	2·0	1·0	180
35 – 44	3·3	2·0	120
45 – 54	7·0	5·0	100
55 – 64	15·0	12·0	70
65 – 74	40·0	35·0	30
75 and above	120·0	110·0	10

Q7. (a) Name any two central statistical organizations in India and explain their functions. *15*

(b) Given the life table of three men A, B and C aged 90, 91 and 92 years respectively as follows :

Age x	l_x
90	16090
91	11490
92	8012
93	5448
94	3607
95	2320
96	1447
97	873
98	590
99	98
100	0

where l_x = Number living at age x.

Find the probability that

(i) A, B and C will be alive in two years time (i.e., at the end of two years)

(ii) All will be dead within two years, and

(iii) C will be alive for 6 years' time. *15*

(c) Explain Autoregressive model and state its stationary invertible property. *10*

Q8. (a) What is logistic curve ? Explain the method of three selected points for fitting the logistic curve to a population data. *15*

(b) Consider the general linear stochastic model :

$$Y = X\beta + U$$

where Y is an $n \times 1$ vector of observations of the dependent variable, X is an observed $n \times k$ matrix of rank k, β is a column vector of k unknown parameters and U is an $n \times 1$ disturbance vector such that

$$E(U) = 0 \text{ and } E(UU') = \sigma^2 I_n.$$

Obtain the expressions for the least squares estimator and its variance – covariance matrix. *15*

(c) Define reliability and validity of a test. Discuss the effect of lengthening of a test on its reliability and validity. How are validity and reliability related to each other ? *10*