

**STATISTICS
Paper—II**

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions :

There are EIGHT questions in all, out of which FIVE are to be attempted.

Question Nos. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Answers must be written in ENGLISH only.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

SECTION—A

- Q. 1 Answer all of the following :—** **8×5=40**
- Q. 1(a) Explain the theoretical basis of control charts. 8
- Q. 1(b) Explain the construction of control charts for number of defectives and fraction defective for detecting lack of control in a continuous flow of manufactured products. 8
- Q. 1(c) Describe single and double sampling inspection plans for attributes. 8
- Q. 1(d) Obtain the steady-state waiting distribution for M/M/K queue. 8
- Q. 1(e) Show that a Markov chain is irreducible if and only if each state can be reached from every other state. 8

Q. 2(a) Obtain Wald's SPRT of strength (α, β) for testing $H_0 : p = p_0$ against $H_1 : p = p_1$ for the distribution :

$$f(x) = p^x (1-p)^{1-x}, \quad x = 0, 1$$

$$= 0, \quad \text{otherwise.}$$

15

Q. 2(b) Write notes on the following :

(i) AOQ

(ii) AOQL

(iii) Dodge-Roming Tables.

5×3=15

Q. 2(c) A manufacturing company has a certain piece of equipment that is inspected at the end of each day and classified as just overhauled, good, fair or inoperative. If the item is inoperative it is overhauled, a procedure that takes one day. Let us denote the four classifications as states 1, 2, 3 and 4 respectively. Assume that the working condition of the equipment follows a Markov chain with the following transition matrix :

$$P = \begin{matrix} & \begin{matrix} \text{Tomorrow} \\ 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} \text{Today} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 3/4 & 1/4 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

If it costs ₹ 125.00 to overhaul a machine (including lost time) on the average and ₹ 75.00 as production lost if a machine is found inoperative. Using the steady-state probabilities, compute the expected per day cost of maintenance. 10

Q. 3(a) The following are the means and ranges of 20 samples of size 5 each of length of a fragmentation bomb base manufactured in a particular war :

Group No.	Mean	Range	Group No.	Mean	Range
1	0.8372	0.010	11	0.8380	0.006
2	0.8324	0.009	12	0.8322	0.002
3	0.8318	0.008	13	0.8356	0.013
4	0.8344	0.004	14	0.8322	0.005
5	0.8346	0.005	15	0.8404	0.008
6	0.8332	0.011	16	0.8372	0.011
7	0.8340	0.009	17	0.8282	0.006
8	0.8344	0.003	18	0.8346	0.006
9	0.8308	0.002	19	0.8360	0.004
10	0.8350	0.006	20	0.8374	0.006

From the data, obtain the control limits for \bar{X} and R-charts to control the length of bomb produced in the future.

(For $n = 5$, $A_2 = 0.58$, $D_3 = 0$ and $D_4 = 2.12$)

15

Q. 3(b) Passengers arriving at a railway reservation office at the rate of 25 per hour. If the approximate average time of issuing ticket is 4 minutes, how many counters should be opened in the least so that the queue does not go on increasing? Supposing that the requisite number of counters are opened, what would be the average waiting time in the queue and the time in which ticket is obtained? 15

Q. 3(c) Explain clearly with suitable examples the different costs that are involved in the inventory problems. 10

Q. 4(a) (i) If the random life time of an item has distribution function $F(x)$, what is the mean remaining life of an item of age x ?

(ii) Find the mean total life time of an item when the distribution function

$$F(x) = 1 - e^{-\lambda x}, \quad x \geq 0, \quad \lambda > 0.$$

Hence show that the mean life time is approximately 2 times the mean life when the renewal process has been in operation for a long time. 10

Q. 4(b) Prove that if the sum of two independent renewal processes is a Poisson process, then both renewal processes must be Poisson processes. 15

Q. 4(c) A dairy firm has three plants located in a state with daily milk production at plant 1, 2 and 3 of 6, 1 and 10 million litres respectively. Each day the firm must fulfil the needs of its four distribution centres 1, 2, 3 and 4 with minimum requirements of 7, 5, 3 and 2 million litres respectively.

Cost in hundreds of rupees of shipping one million litre from each plant to each distribution centre is given in the following table :

		Distribution centre			
		D ₁	D ₂	D ₃	D ₄
Plant	P ₁	2	3	11	7
	P ₂	1	0	6	7
	P ₃	5	8	15	9

Find initial basic feasible solution for this problem using :

(i) North-West corner rule

(ii) Least cost method and

(iii) Vogel's approximation method. 15

SECTION—B

Q. 5 Answer all of the following :—

8×5=40

Q. 5(a) Describe Greville's method of constructing abridged life table. 8

Q. 5(b) Describe the main sources of demographic data in India. 8

Q. 5(c) Give a brief outline of factor analysis and discuss its importance in psychometric analysis. 8

- Q. 5(d) Describe the classical linear regression model. If the disturbances are independently and normally distributed, show that the OLS method and the ML method provide identical estimators for the regression co-efficients. 8
- Q. 5(e) Briefly describe any two tests for detecting multicollinearity. 8
- Q. 6(a) Discuss the different methods of determining trend in a time series. What are their relative merits and demerits ? 15
- Q. 6(b) Construct with the help of data given below Fisher's ideal index number and show that it satisfies the factor reversal test :

Commodity	Estimated total produce in quintals by a small factory		Price per quintal in Rs.	
	2001	2005	2001	2005
Commodity I	71	26	350	312
Commodity II	107	83	200	187
Commodity III	62	48	256	175

- Q. 6(c) Describe the merits of using standardised rates in place of crude rates. Explain why would you consider standardised death rates to give a better measure for comparison of mortality situation of two communities. 10
- Q. 7(a) Explain different stages of Box Jenkins method of forecasting. 15
- Q. 7(b) Write note on the following tests for index numbers :
 (i) Circular test
 (ii) Time reversal test
 (iii) Factor reversal test. 5×3=15
- Q. 7(c) Fill in the blanks in a portion of life table given below :

Age in Years	l_x	d_x	P_x	q_x	L_x	T_x	e_x^0
4	95,000	500	?	?	?	48,50,300	?
5	?	400	?	?	?	?	?

Show details of computation.

- Q. 8(a) Explain the different methods of combining and comparing scores in several tests in psychology. 10
- Q. 8(b) What do you mean by intelligence quotient ? Describe the procedure and tests for measuring intelligence quotient. 15
- Q. 8(c) Discuss various official statistics of India relating to Industry. 10