STATISTICS

Paper – I

*Time Allowed : **Three** Hours*

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) An urn U contains 3 white and 4 black balls and another urn V contains 5 white and 3 black balls. One ball is chosen at random from urn U and transferred to urn V. If two balls are chosen without replacement from V, what is the probability that they are of the same colour ? *8*

(b) Let X be a random variable with the following probability density function :

$$f(x) = \begin{cases} \frac{x}{2}, & 0 \le x < 1 \\ \frac{1}{2}, & 1 \le x < 2 \\ \frac{3-x}{2}, & 2 \le x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the cumulative distribution function of X and hence obtain the value of $P(X > \frac{3}{2})$. *8*

(c) (i) Define an unbiased estimator of a population parameter θ. Give an example to show that unbiased estimator is not unique. *5*

(ii) Let $X_1, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Check whether $\overline{X}$ is an ancillary statistic with respect to parameter σ^2. *3*

(d) (i) The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers, five or six will catch the disease ? *4*

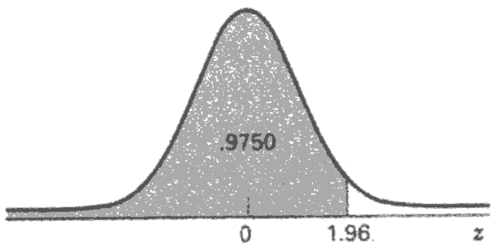
(ii) Suppose $X_1, X_2, \ldots$ are independent and identically distributed random variables from $B(1, p)$. Find a consistent estimator of p. *4*

(e) Let $X_1, \ldots, X_n$ be independent $N(\mu, \sigma^2)$ random variables with known σ^2. Using Monotone Likelihood Ratio Approach, find a Uniformly Most Powerful (UMP) test for testing

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu > \mu_0.$$

Write the UMP test for $n = 5$, $\sigma^2 = 9$, $\alpha = \cdot 01$ and $\mu_0 = 5$.
(Tables are provided.) *8*

Normal Distribution Table



z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00	z
−3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.80
−3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.70
−3.60	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	−3.60
−3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	−3.50
−3.40	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	−3.40
−3.30	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	−3.30
−3.20	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	−3.20
−3.10	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	−3.10
−3.00	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	−3.00
−2.90	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	−2.90
−2.80	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	−2.80
−2.70	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	−2.70
−2.60	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	−2.60
−2.50	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	−2.50
−2.40	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	−2.40
−2.30	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	−2.30
−2.20	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	−2.20
−2.10	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	−2.10
−2.00	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	−2.00
−1.90	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	−1.90
−1.80	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	−1.80
−1.70	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	−1.70
−1.60	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	−1.60
−1.50	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	−1.50
−1.40	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	−1.40
−1.30	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	−1.30
−1.20	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	−1.20
−1.10	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	−1.10
−1.00	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	−1.00
−0.90	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	−0.90
−0.80	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	−0.80
−0.70	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420	−0.70
−0.60	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	−0.60
−0.50	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	−0.50
−0.40	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	−0.40
−0.30	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	−0.30
−0.20	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	−0.20
−0.10	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	−0.10
0.00	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	0.00

Normal Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	0.00
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	0.10
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	0.20
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	0.30
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	0.40
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	0.50
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	0.60
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	0.70
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	0.80
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	0.90
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	1.00
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	1.10
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	1.20
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	1.30
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1.40
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1.50
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1.60
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1.70
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1.80
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1.90
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	2.00
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	2.10
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	2.20
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	2.30
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	2.40
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	2.50
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	2.60
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	2.70
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	2.80
2.90	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	2.90
3.00	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	3.00
3.10	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	3.10
3.20	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	3.20
3.30	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	3.30
3.40	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	3.40
3.50	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	3.50
3.60	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.60
3.70	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.70
3.80	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.80

Q2. (a) Let $\{A_n, n \geq 1\}$ be a sequence of events on a probability space $(\Omega, \mathcal{A}, P)$ and let $F_n = \bigcup_{k=n}^{\infty} A_k$, for $n \geq 1$. Then show that

$$P(\liminf A_n) \leq \lim_{n \to \infty} P(F_n).$$ *10*

(b) Let $\{A_n, n \geq 1\}$ be a sequence of events on a probability space $(\Omega, \mathcal{A}, P)$. Prove or disprove :

$$P(\limsup A_n) \leq \lim_{n \to \infty} \sum_{k=n}^{\infty} P(A_k)$$ *10*

(c) (i) Let X_1 and X_2 be independent and identically distributed as Poisson with parameter λ. For $\alpha > 1$ (an integer), check for sufficiency of the estimator $T = X_1 + \alpha X_2$ for parameter λ. *5*

(ii) Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$. Show that $T = \max(X_1, ..., X_n)$ is complete for θ. *8*

(iii) Let $X_1, ..., X_n$ be independent and identically distributed (iid) $N(\mu, \sigma^2)$, with both μ and σ^2 unknown. Find a minimal sufficient statistic for (μ, σ^2). *7*

Q3. (a) Suppose that $X_1, X_2, ..., X_8$ are observed as

1·43, 0·25, 1·99, 0·40, 0·58, 0·78, 1·5, 1·2.

Using Kolmogorov – Smirnov's goodness-of-fit test, check whether the data were randomly drawn from a $U(0, 2)$ distribution.

(Critical value is 0·46 for $\alpha = 0{\cdot}05$) *10*

(b) (i) Describe Wald's Sequential Probability Ratio Test (SPRT). *6*

(ii) Let $X_1, ..., X_n$ be independent and identically distributed as $N(\mu, \sigma^2)$, where σ^2 is known. Write the $100(1 - \alpha)\%$ UMA lower confidence bound for testing

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu > \mu_0.$$ *4*

(c) (i) Consider a sequence $\{X_k, k \geq 1\}$ of independent and identically distributed random variables, each with finite mean μ. For $k \geq 1$, let $T_k = X_1 + X_2 + \ldots + X_k$ and $Y_k = \frac{T_k - k\mu}{k}$. Show that Y_k converges in distribution to a random variable Y with $P(Y = 0) = 1$ as k tends to infinity. *12*

(ii) Three coins whose faces are marked 1 and 2, are tossed. Find the mean and variance of total value of numbers on their faces. *8*

Q4. (a) (i) Let $X_1, \ldots, X_n$ be a random sample from $U(\theta_1, \theta_2)$. Find the maximum likelihood estimators of θ_1 and θ_2. *6*

(ii) Suppose 47, 40, 50, 14, 30, 25, 37, 60 is a random sample from discrete uniform distribution with probability mass function as

$$P(X = x) = \frac{1}{N}, \; x = 1, 2, \ldots, N.$$

Find the value of estimate of N using the method of moments. *4*

(b) (i) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space {1, 2, 3, 4} and the following transition probability matrix :

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

(1) Which states of this Markov chain are recurrent and which states are transient ?

(2) Obtain the mean recurrence time in case of these identified recurrent states.

(3) Examine whether these identified recurrent states are ergodic. Justify. *5+5+5=15*

(ii) Consider a Markov chain with state space {1, 2, 3} and transition probability matrix $P = ((p_{ij}))$ such that the probabilities corresponding to cells (1, 2) and (1, 3) are equal; (2, 1) and (3, 1) are equal and all other cells have zero probability. If $p_{jj}^{(n)}$ denotes the n-step transition probability of state j, compute $p_{11}^{(9)}$ and $p_{11}^{(98)}$. *5*

(c) (i) Let $X_1, ..., X_n$ follow Poisson distribution with parameter θ and let

$$g(\theta) = e^{-\theta}(1 + \theta).$$

Find the Cramer – Rao (CR) lower bound for estimator of $g(\theta)$. *6*

(ii) Let X follow B(n, p) and let the loss function be

$$L(p, \delta(x)) = [p - \delta(x)]^2.$$

If $\pi(p) = 1, \quad 0 < p < 1$

is the prior pdf of p, then find Bayes' estimator of $\delta(X)$. *4*

SECTION B

Q5. (a) What is curvilinear regression and when is it used ? Give some examples of curvilinear regression. How will you use scatter plot and adjusted R-squared value to identify curvilinear regression ? *8*

(b) If X_1, X_2 and X_3 are uncorrelated variables each having zero mean and the same standard deviation, then show that the correlation coefficient between $(X_1 + X_2)$ and $(X_2 + X_3)$ is equal to (1/2). Why is the correlation coefficient not zero ?

Further if we let $Y_1 = X_1 + X_2$, $Y_2 = X_2 + X_3$ and $Y_3 = X_3 + X_1$, then obtain :

(i) Partial correlation coefficient $r_{y_1 y_2 \cdot y_3}$,

(ii) Multiple correlation coefficient $R^2_{y_1 \cdot y_2 y_3}$.

Is there any relation between partial and multiple correlation coefficients you have obtained ? *8*

(c) Let $\mathbf{x} \sim N_3(\mu, \Sigma)$ where $\boldsymbol{\mu}' = (1, -1, 2)$ and

$$\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

Obtain the conditional distributions of :

(i) X_1, given $X_3 = x_3$, and

(ii) X_1, X_2, given $X_3 = x_3$. *8*

(d) Derive and compare the variances of the ratio and regression estimators of the population mean and state when the ratio estimator is as good as the regression estimator. *8*

(e) A randomized block design gave the following results :

	Degrees of Freedom	Sum of Squares
Blocks	5	620·4
Treatments	3	415·03
Error	11	121·50

How many replications per treatment are required for a completely randomized design to be as effective as the randomized block design using the same experimental material ? *8*

Q6. (a) Explain the concept of interval estimation for the parameters of linear regression model

$$y_i = \beta_1 + \beta_2 X_i + u_i.$$

For the weekly consumption expenditure and weekly income data, the following results were obtained :

$$\hat{\beta}_1 = 24{\cdot}4545,\ \text{Var}(\hat{\beta}_1) = 41{\cdot}1370 \text{ and } se(\hat{\beta}_1) = 6{\cdot}4138$$

$$\hat{\beta}_2 = 0{\cdot}5091,\ \text{Var}(\hat{\beta}_2) = 0{\cdot}0013 \text{ and } se(\hat{\beta}_2) = 0{\cdot}0357$$

$$\text{Cov}(\hat{\beta}_1, \hat{\beta}_2) = -0{\cdot}2172,\ \hat{\sigma}^2 = 42{\cdot}1591$$

$$r^2 = 0{\cdot}9621,\ r = 0{\cdot}9809,\ df = 8$$

Write estimated linear regression model. Obtain 95 percent confidence intervals for $\hat{\beta}_1$ and $\hat{\beta}_2$. Interpret the results.

$(t_{\alpha/2} = t_{0{\cdot}025} = 2{\cdot}306$ for 8 df$)$ *15*

(b) Find the principal components and the proportion of the total variance explained by each principal component of the standardized covariance matrix, when the covariance matrix

$\Sigma = (\sigma_{ij})$; $\sigma_{ii} = \sigma^2$ for i = 1, 2, 3, 4 and $\sigma_{ij} = \rho\sigma^2$

for $i \neq j$; i, j = 1, 2, 3, 4. *15*

(c) Consider the data matrix :

$$X = \begin{pmatrix} -1 & 3 & -2 \\ 2 & 4 & 2 \\ 5 & 2 & 3 \end{pmatrix}$$

(i) Calculate the matrix of deviations (residuals), $(X - \mathbf{1}\bar{\mathbf{x}}')$. Is this a non-singular matrix ?

(ii) Determine $S = \frac{1}{n-1}(X - \mathbf{1}\bar{\mathbf{x}}')' (X - \mathbf{1}\bar{\mathbf{x}}')$ and calculate the generalised sample variance $|S|$.

(iii) Calculate the total sample variance. *10*

Q7. (a) (i) Define the Horvitz Thompson (HT) estimator. In a PPSWOR sampling, show that the HT estimator is unbiased for the population total.

(ii) Calculate the inclusion probability for the following population where x is the size variable when two units are selected without replacement with PPS at each step. *5+10=15*

S.No.	y_i	x_i
1	2	1
2	6	2
3	8	7
4	24	10

(b) (i) How are the three principles of experimentation implemented in a Latin square ?

(ii) A scientist needed to test the strength of four different types of polymers A, B, C, D and a control E. He made five samples of each polymer. He suspected there was a temperature gradient from front to back and top to bottom in the thermal furnace. He conducted the experiment using shelves as rows, and position in shelves as columns to obtain the following data. Analyze the data and give your conclusion on his belief at both 5% and 1% significance level. *5+10=15*

	Front →				Back
Top ↓	33·8 (A)	33·7 (B)	30·4 (D)	32·7 (C)	24·4 (E)
	37·0 (D)	28·8 (E)	33·5 (B)	34·6 (A)	33·4 (C)
	35·8 (C)	35·6 (D)	36·9 (A)	26·7 (E)	35·1 (B)
	33·2 (E)	37·1 (A)	37·4 (C)	38·1 (B)	34·1 (D)
Bottom	34·8 (B)	39·1 (C)	32·7 (E)	37·4 (D)	36·4 (A)

[Tables are provided]

F-distribution Table

$F_{0.95}$

Denominator Degrees of Freedom	Numerator Degrees of Freedom								
	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

F-distribution Table

Denominator Degrees of Freedom	$F_{0.99}$ Numerator Degrees of Freedom									
	10	12	15	20	24	30	40	60	120	∞
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

(c) A farmer packing pineapples used 600 baskets to collect the fruit from all his farms. Each basket could hold 40 fruits. But some baskets held lesser number of the fruits. A simple random sample of 50 baskets was examined and the data are given below :

No. of fruits	40	39	36	32	29	27	23	19			
No. of baskets	21	3	4	1	1	1	2	1			
No. of fruits	16	15	14	11	10	9	7	6	5	4	3
No. of baskets	1	2	2	1	1	1	1	3	2	1	1

Give the 80% confidence interval for the total number of pineapples he has harvested from his farms. (Use table value t = 1·28, if required) *10*

Q8. (a) Discuss all the assumptions and their relative importance for Classical Linear Regression Model (CLRM) in the context of the two-variable linear regression model. *15*

(b) (i) Consider two factors A and B at three levels. Under a 3^2 factorial experiment, give the contrast for estimating the linear and quadratic effects of A and B and the interaction effects.

(ii) Give the assignment of treatments, along with breakdown of the total degrees of freedom, in a 3^3 factorial experiment, into blocks by confounding the interaction effect AB^2. *5+10=15*

(c) A rural block in a district is divided into three strata comprising 10, 8 and 25 villages, respectively. The following table gives the number of villages, area of cultivation of pepper and standard deviation of area under cultivation of pepper for different strata.

Stratum No.	Number of Villages	Area Under Cultivation (hectares)	Standard Deviation
1	10	425	90·5
2	8	470	40·7
3	25	855	100·8

A stratified sample of 10 villages is selected and yields are given below :

Stratum No.	Yield in quintals for the selected villages				
1	8·5	10·2			
2	7·6	9·3	8·5		
3	7·4	10·1	8·3	8·5	8·9

(i) Give an unbiased estimate of average yield based on the above stratified design.

(ii) Do you think that the stratification is more effective than a simple random sampling in this case ? *10*