PHYSICS

Paper - I

*Time Allowed : **Three** Hours*

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Useful Constants :

Electron charge (e)	$= 1{\cdot}602 \times 10^{-19}$ C
Electron rest mass (m_e)	$= 9{\cdot}109 \times 10^{-31}$ kg
Proton mass (ni_p)	$= 1{\cdot}672 \times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8{\cdot}854 \times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1{\cdot}257 \times 10^{-6}$ henry/m
Velocity of light in free space (c)	$= 3 \times 10^{8}$ m/s
Boltzmann constant (k)	$= 1{\cdot}380 \times 10^{-23}$ J/K
Electron volt (eV)	$= 1{\cdot}602 \times 10^{-19}$ J
Planck constant (h)	$= 6{\cdot}626 \times 10^{-34}$ Js
Stefan constant (σ)	$= 5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4}
Avogadro number (N)	$= 6{\cdot}022 \times 10^{26}$ kmol^{-1}
Gas constant (R)	$= 8{\cdot}31 \times 10^{3}$ J kmol^{-1} K^{-1}
exp (1)	$= 2{\cdot}718$

SECTION A

Q1. **Answer the following :** *8×5=40*

(a) A bead slides on a wire in the shape of a cycloid described by the equations

$$x = a(\theta - \sin\theta)$$

$$y = a(1 + \cos\theta) \quad \text{with} \quad 0 \le \theta \le 2\pi.$$

Find the Lagrangian and equation of motion. *8*

(b) Show that the relativistic invariance laws of conservation of momentum lead to the concepts of variation of mass with velocity and mass energy equivalence. *8*

(c) A parallel beam of light of wavelength 5890 Å is incident at an angle of 30° on a plane transmission grating with 15000 lines/inch. Find the highest order of spectrum that can be observed. *8*

(d) Discuss absorption loss in an optical fibre comparing and contrasting the intrinsic and extrinsic absorption mechanisms. *8*

(e) Although the principle of operation of a basic LASER is based upon two energy levels, why does one need a 3-level or a 4-level scheme to achieve satisfactory lasing ? Explain your answer with special reference to a Ruby-laser. *8*

Q2. (a) Discuss the mechanics of a system of point particles with special emphasis on the conservation theorems. How can we extend the results to a system with continuous mass distribution ? *10*

(b) (i) State and prove Hamilton's principle and use it to prove that the shortest distance between two points in space is a straight line joining them.

(ii) Use Hamiltonian mechanics to find the differential equation for planetary motion, moving under force $f(r) = -\frac{k}{r^2}$ and prove that the areal velocity is constant. *8+7=15*

(c) (i) What is Holography ?

(ii) Show with simple diagrams, how a hologram is written and read using a laser.

(iii) Mention some important applications of holography. *3+8+4=15*

Q3. (a) State the fundamental postulates of Einstein's special theory of relativity. Deduce Lorentz transformation equation and discuss how this accounts for the phenomenon of length contraction. *10*

(b) Discuss the properties of Cornu spiral. Show that the spiral can be used to obtain the intensity distribution in the Fresnel's diffraction pattern due to a straight edge. *10*

(c) (i) Using the concept of spontaneous and stimulated emission of radiation, obtain the relation between Einstein's A and B coefficients.

(ii) What is the physical significance of Einstein's A coefficient ?

(iii) Justify why lasing action is much more difficult at X-ray frequency than in case of infrared frequency spectrum. *10+5+5=20*

Q4. (a) (i) In a Michelson's interferometer, 100 fringes cross the field of view when the movable mirror is displaced through $2{\cdot}894 \times 10^{-3}$ cm. Calculate the wavelength of the monochromatic source of light.

(ii) A shift of 200 fringes is observed when the movable mirror of a Fabry-Pérot interferometer is shifted by 0·0298 mm. Calculate the wavelength of the incident radiation. *8+7=15*

(b) State and explain Fermat's principle of extremum path and use the same to deduce the laws of reflection and refraction of light. *10*

(c) (i) Explain the reason for pulse broadening due to intermodal and material dispersion. Deduce the relation of pulse broadening for intermodal dispersion in optical fiber.

(ii) A step index fiber in air has a numerical aperture of 0·16, a core refractive index of 1·45 and a core diameter of 60 μm. Determine the normalized frequency for the fiber when light at a wavelength of 0·8 μm is transmitted. Also estimate the number of guided modes propagating in the fiber. *10+5=15*

SECTION B

Q5. Answer the following : *8×5=40*

(a) In a one-dimensional device, the charge density is given by

$$\rho_V = \rho_0 \frac{x}{a}.$$

If $E = 0$ at $x = 0$ and $V = 0$ at $x = a$,
find V and E using Laplace equation of electrostatics. *8*

(b) State and explain the Biot-Savart law. Derive an expression for the magnetic field at a point due to an infinitely long straight current carrying conductor. *8*

(c) Write down the four Maxwell's equations and explain the contribution of Maxwell in the development of these equations. *8*

(d) Prove the thermodynamic relation :

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

and hence show that

$$\frac{dP}{dt} = \frac{L}{T(V_2 - V_1)};$$

all the terms have their usual meanings. *8*

(e) Describe neutron star on the basis of Fermi-Dirac statistics and obtain the condition of critical mass for a neutron star. *8*

Q6. (a) Use the method of electric images to find the electric field on the surface of a grounded conducting sphere. *10*

(b) (i) State Faraday's law of electromagnetic induction and prove that it can be expressed in the following vector form :

$$\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

with $\vec{E}$ and $\vec{B}$ being the electric and magnetic fields.

(ii) A coil of 10 turns has dimension 9 cm × 7 cm. It rotates at the rate of 15π rad/sec in a uniform field whose flux density is 0·6 weber/m^2. What is the maximum e.m.f. induced in the coil ? *10+5=15*

(c) How does one explain the observed spectrum of black-body radiation using Planck's quantum hypothesis ? State and obtain Wien's displacement law. Also explain the important features of this law. *15*

Q7. (a) (i) Using Maxwell's equations, obtain the relation

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{E^2 + B^2}{2}\right) + \overline{\nabla}\cdot(\overline{E} \times \overline{B}) = 0$$

(ii) What is Poynting vector ? Deduce Poynting theorem for the flow of energy in an electromagnetic field. *5+10=15*

(b) Discuss the reflection and refraction of plane electromagnetic waves at plane dielectric boundaries for normal incidence and also find the reflection and transmission coefficients. *15*

(c) What do you understand by spontaneous magnetization below Curie temperature ? Explain with an appropriate diagram, the occurrence of a hysteresis loop in a ferromagnetic material. *10*

Q8. (a) State Maxwell's distribution law of molecular speeds. Draw and explain a curve between n(c) and c in a gas at a given temperature T, where n(c) dc is the number of molecules having speed between c and c + dc. Discuss the effect of T and mass m of the molecule on the nature of the curve. *15*

(b) (i) Define and explain the significance of the quality factor of an electrical machine.

(ii) Discuss in brief, the working principle of a transformer. *5+5=10*

(c) Derive the mathematical expression for the total energy of a degenerate Fermi gas at a temperature T and calculate the specific heat of the Fermi gas at this temperature. *15*