I.E.S/ I.S.S. Examination, 2024

STATISTICS

Paper – IV

Time Allowed : ***Three*** *Hours* | *Maximum Marks :* **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are ***FOURTEEN*** *questions divided under* ***SEVEN*** *Sections.*

Candidate has to choose any ***TWO*** *Sections and attempt the questions therein. All the Sections carry equal marks.*

The number of marks carried by a question / part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams / Figures, wherever required, shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in ***ENGLISH*** *only.*

SECTION A
(Operations Research and Reliability)

Q1. (a) An electronics company manufactures two types of LEDs (Model 1 and Model 2). The daily capacity of the company is 60 LEDs of Model 1 and 75 LEDs of Model 2. Each model of the first type uses 10 pieces of an electronic component whereas a model of second type uses 8 pieces of this component. The maximum availability of this component is 800 pieces. The company is committed to manufacture at least 20 pieces of Model 2 per day. If the net profit on the sale of these two models is ₹ 40 and ₹ 30 respectively, then determine the production schedule which maximizes the profit of the company. (Graph paper is attached) *10*

(b) Define saddle point of a game problem. Use the Principle of Dominance to find the optimal strategies of the two players with the following payoff matrix : *10*

Player B

$$\text{Player A} \quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 3 & 5 & 4 & 2 \\ 5 & 6 & 2 & 4 \\ 2 & 1 & 4 & 0 \\ 3 & 3 & 5 & 2 \end{pmatrix} \end{array}$$

(c) Use Vogel's Approximation Method to find feasible solution to the following transportation problem : *10*

Destinations

		D_1	D_2	D_3	D_4	Supply
Origins	O_1	2	2	2	1	3
	O_2	10	8	5	4	7
	O_3	7	6	6	8	5
Requirement		4	3	4	4	

(d) Let $\phi(\mathbf{x})$ be a coherent structure of n components. Show that, under usual notations,

$$\phi(\mathbf{x} \amalg \mathbf{y}) \geq \phi(\mathbf{x}) \amalg \phi(\mathbf{y}).$$

What is the application of this principle among engineers ? *10*

(e) A non-negative random variable T has cumulative distribution function F(t) given by :

$$F(t) = 1 - \frac{8}{7}e^{-t} + \frac{1}{7}e^{-8t};\ t > 0$$

Find the hazard rate and mean time to failure. *10*

Q2. Answer any ***two*** from the following :

(a) (i) We have five jobs A, B, C, D and E. Each of these jobs must go through three machines M_1, M_2 and M_3 for processing in the order $M_1M_2M_3$. Processing time (in hours) are given below :

Processing Time (in hours)

Jobs	M_1	M_2	M_3
A	4	5	8
B	9	6	10
C	8	2	6
D	6	3	7
E	5	4	11

Determine an optimal sequence of jobs that will minimize the total elapsed time. Also, calculate the idle times of the three machines. *20*

(ii) Explain Hungarian Method for finding optimal solution to an Assignment Problem. *5*

(b) (i) Consider the following project :

Activity	Time (in days)
(1, 2)	4
(1, 3)	3
(2, 4)	6
(3, 5)	5
(4, 5)	7
(4, 6)	9
(5, 7)	8

Find the critical path and duration of completion of the above project. *15*

(ii) An industry needs 5400 units/year of a bought out component which will be used in its main product. The ordering cost is ₹ 250 per order and carrying cost per unit/year is ₹ 30. Find the Economic Order Quantity (EOQ), the number of orders/year and the time between successive orders. *10*

(c) Let T denote the random variable having Weibull distribution with edf F(t) given by :

$$F(t) = 1 - e^{-\lambda t^{\alpha}};\ t, \alpha, \lambda > 0.$$

(i) Discuss the monotonicity of the hazard rate. *10*

(ii) Describe the procedure for finding the MLEs of its parameters, by constructing the likelihood function, using a time censored sample. *15*

(d) (i) Show that the hazard rate of a series system of n independent components is the sum of its component hazard rates. Hence, show that if all components have IFR, then the series system is also of the IFR type. *15*

(ii) Consider the following linear programming problem :

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to

$$\frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 \leq 1 \text{ (labour)}$$

$$\frac{1}{3}x_1 + \frac{4}{3}x_2 + \frac{7}{3}x_3 \leq 3 \text{ (material)}$$

$$x_1, x_2, x_3 \geq 0$$

with optimal table as

C_b	X_b \ C_j	2 x_1	3 x_2	1 x_3	0 x_4	0 x_5	b
2	x_1	1	0	− 1	4	− 1	1
3	x_2	0	1	2	− 1	1	2
	$Z_j - C_j$	0	0	3	5	1	Z = 8

Determine, how far the availability of labour can be varied such that the optimal table remains optimal. *10*

SECTION B
(Demography and Vital Statistics)

Q3. (a) Define the force of mortality at exact age x. Derive its formula. If $l(x) = 100\sqrt{100 - x}$, using an appropriate method find $\mu(84)$. *10*

(b) Discuss various types of migration. Explain Vital Statistical Method of obtaining Net Migration (NM). *10*

(c) Define Stable and Stationary population. Discuss their properties. Write the importance of the Stable Population theory in the study of Vital Statistics. *10*

(d) Write the salient features of Indian Census 2011. Also mention the importance of preparing census. *10*

(e) Explain Gompertz's law for mortality, and derive the formula for the law. Also write its applications. *10*

Q4. Answer any ***two*** from the following :

(a) Discuss the different measures of mortality with their relative merits and demerits. *25*

(b) Define life table and write its importance in the study of Vital Statistics in detail. Discuss various measures of life table functions. *25*

(c) Define Age Specific Fertility Rate. Write its importance in Vital Statistics. For the following data, calculate ASFR. For which age group is ASFR the highest ? Is a fertility curve of ASFR against age positively skewed ?

Age of Mother	Number of live births	Number of females ('000)
15 – 19	23,604	5,467
20 – 24	4,01,080	4,520
25 – 29	9,48,848	4,371
30 – 34	3,84,315	4,210
35 – 39	77,871	3,899
40 – 44	9,489	3,460
45 – 49	435	2,850

Also calculate GFR and GRR when the proportion of female births is 46·5 percent. *25*

(d) Explain the method due to Rhodes for fitting of Logistic curve for population projection. Illustrate how you would fit a logistic curve to the given data set using this method. *25*

SECTION C
(Survival Analysis and Clinical Trials)

Q5. (a) Define time, order and random censoring schemes. Give an example of each. *10*

(b) The following data relates to survival times of patients after a surgery. (+ denotes right censored observations)

Male :	6+,	11,	13+,	15,	16+
Female :	2+,	4,	7,	8+	

A Cox proportional model $\lambda(t, x) = \lambda_0(t)e^{\beta x}$ is to be fitted to these data; where

$\lambda_0(t)$ is baseline hazard
$x = 0$ if male
$\quad = 1$ if female
t is the surviving time after surgery

Construct Cox partial likelihood for the data. *10*

(c) Explain competing risks model in survival analysis. *10*

(d) Distinguish between controlled and uncontrolled clinical trials. *10*

(e) Describe multicenter trials in clinical trials. *10*

Q6. Answer any ***two*** from the following :

(a) Describe Gamma distribution as a failure time model in survival analysis. Explain why this distribution is less popular compared to Weibull distribution. Write down the likelihood in the order censored sample. *25*

(b) In a study, 30 patients are observed for two years following treatment with a new drug. The following data shows the period t in complete months from the initial treatment to the end of the observation for those patients who died or withdrew from the trial before the end of the two year period.

Deaths :	6,	6,	12,	15,	20,	20,	23.
Withdrawals :	1,	3,	5,	8,	10,	18.	

(i) Calculate Kaplan-Meier estimate $\hat{S}(t)$ of S(t). *15*

(ii) Calculate $\hat{V}(\hat{S}(t))$ at t = 14. *10*

(c) Describe the ethics, objectives and protocols to be followed in conducting the clinical trials. *25*

(d) (i) Describe data collection system in clinical trials. *10*

(ii) Explain parallel and cross-over designs in clinical trials. *15*

SECTION D
(Quality Control)

Q7. (a) Distinguish between process control and product control. Also explain how the chance causes and assignable causes affect both. *10*

(b) The following data gives the number of 10 independent samples of varying size from a production process :

Sample number	Sample size	Number of defectives
1	2000	425
2	1500	430
3	1400	216
4	1350	341
5	1250	225
6	1760	322
7	1875	280
8	1955	306
9	3125	337
10	1575	305

Draw the control chart for fraction defective and comment on it. (A graph paper is provided to draw the chart) *10*

(c) Discuss the use and importance of control chart for number of defects per unit. Suppose a manufacturing process results in 2% defects per unit. Obtain the control limits for c-chart if 500 units are sampled. *10*

(d) Explain the procedure of a single sampling inspection plan and obtain its operating characteristic function. Suppose large batches of screws are subjected to a single sampling plan with n = 60 and c = 2. If the process average is $\overline{p} = 0{\cdot}01$, does this lot accept batch of high quality with high probability ? *10*

(e) Explain the following terms with respect to sampling inspection plans :

(i) Acceptance Quality Level (AQL)

(ii) Lot Tolerance Percent Defective (LTPD)

(iii) Average Outgoing Quality Limit (AOQL)

(iv) Operating Characteristic Curve

(v) Consumer's Risk *10*

Q8. Answer any ***two*** from the following :

(a) The following data were collected from a process manufacturing power supplies company. The variable of interest is output voltage and n = 5.

Sample Number	$\bar{x}$	R	Sample Number	$\bar{x}$	R
1	103	4	11	105	4
2	102	5	12	103	2
3	104	2	13	102	3
4	105	8	14	105	4
5	104	4	15	104	5
6	106	3	16	105	3
7	102	7	17	106	5
8	105	2	18	102	2
9	106	4	19	105	4
10	104	3	20	103	2

(i) Compute center line and control limits for controlling future production. (Use $\bar{x}$-chart and R-chart)

(ii) Assuming that the quality characteristic is normally distributed, estimate the process standard deviation.

(iii) What are the natural tolerance limits of the process ?

(iv) Estimate the process fraction non-conforming if the specifications were 103 ± 4.

(v) Obtain the capability (both C_p and C_{pk}) of the process.

(Given for n = 5, $A_2 = 0{\cdot}58$, $D_3 = 0$, $D_4 = 2{\cdot}115$, $d_2 = 2{\cdot}326$) *25*

(b) Explain the utility and applications of Cumulative Sum (CUSUM) chart. A machine produces spokes for the wheels of bicycles. When the process is in control, the machine produces spokes whose lengths are normally distributed about mean 25 cm and standard deviation 0·02 cm. The sample values are given as follows :

24·998	25·016	25·024	25·022	24·984
25·012	25·024	24·978	25·012	25·017
25·024	25·026	25·027	25·028	25·028

Use CUSUM chart to investigate whether the machine is correctly set. Also investigate at what point the linearity is observed and provide the corresponding estimate. Plot the chart in the graph paper provided. 25

(c) What is Acceptance Sampling Plan ? In usual notation, explain the procedure of Double Sampling Plan. Also state how it differs from sequential sampling plan. 25

(d) The following data represents individual observations on molecular weight taken hourly from a chemical process :

1045	1055	1037	1064	1095	1008	1050
1087	1125	1146	1139	1169	1151	1128
1238	1125	1163	1188	1146	1167	

The target value of molecular weight is 1050 and the process standard deviation is thought to be about 25. Use an Exponentially Weighted Moving Average (EWMA) chart with $\lambda = 0{\cdot}2$ and $L = 2{\cdot}7$ and analyse the data. A manual chart may be prepared to show the steady-state position. 25

SECTION E
(Multivariate Analysis)

Q9. (a) Energy consumption in 2001, by State, from the major sources

x_1 = petroleum

x_2 = natural gas

x_3 = hydroelectric power

x_4 = nuclear electric power

is recorded in quadrillions (10^{15}) of BTUs.

The resulting mean and covariance matrix are :

$$\bar{x} = \begin{bmatrix} 0{\cdot}766 \\ 0{\cdot}508 \\ 0{\cdot}438 \\ 0{\cdot}161 \end{bmatrix}, \; S = \begin{bmatrix} 0{\cdot}856 & 0{\cdot}635 & 0{\cdot}173 & 0{\cdot}096 \\ 0{\cdot}635 & 0{\cdot}568 & 0{\cdot}128 & 0{\cdot}067 \\ 0{\cdot}173 & 0{\cdot}127 & 0{\cdot}171 & 0{\cdot}039 \\ 0{\cdot}096 & 0{\cdot}067 & 0{\cdot}039 & 0{\cdot}043 \end{bmatrix}$$

Using the summary statistics, determine the sample mean and variance of the State's total energy consumption for these major sources when $\mathbf{x} : 4 \times 1$ is distributed as $N_4(\mu, \Sigma)$. *10*

(b) Let $\mathbf{x}_r$, r = 1, 2, ..., k be independently identically distributed as $N_p(\mu, \Sigma)$. For fixed matrices $A_r : m \times p$, r = 1, 2, ..., k, obtain the distribution of

$$\sum_{r=1}^{k} A_r \mathbf{x}_r .$$

Using the distribution you have obtained, derive the distribution of the sample mean vector $\bar{\mathbf{x}}$. *10*

(c) Let $\mathbf{x} \sim N_3(\mu, \Sigma)$ where $\mu' = (1, -1, 1)$ and

$$\Sigma = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}.$$

Obtain the joint distribution of $y_1 = 2x_1 + 2x_2 - 3x_3$ and $y_2 = x_1 - x_2 + 3x_3$. Obtain the correlation coefficient between y_1 and y_2. Are they positively correlated ? *10*

(d) Define Wishart matrix and Wishart distribution. If $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ are independent observations from $N_2(\boldsymbol{\mu}, \Sigma)$ population with $\boldsymbol{\mu} = \mathbf{0}$ and $\Sigma = \begin{pmatrix} 1 & 2 \\ 2 & 9 \end{pmatrix}$, then

(i) compute $E(\mathbf{x}_1\mathbf{x}_1' + \mathbf{x}_2\mathbf{x}_2' + \mathbf{x}_3\mathbf{x}_3')$

(ii) compute $E(|\mathbf{x}_1\mathbf{x}_1' + \mathbf{x}_2\mathbf{x}_2' + \mathbf{x}_3\mathbf{x}_3'|)$

(iii) mention the distribution of $(\mathbf{x}_1\mathbf{x}_1' + \mathbf{x}_2\mathbf{x}_2' + \mathbf{x}_3\mathbf{x}_3')$. *10*

(e) Define canonical correlations and variates. State the condition when (i) multiple correlation coefficient, and (ii) simple correlation coefficient can be obtained as a particular case of canonical correlations.

In usual notations, show that, if λ_i is an eigenvalue of

$$\Sigma_{11}^{-\frac{1}{2}} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11}^{-\frac{1}{2}}$$

with associated eigenvector $\mathbf{e}_i$, then λ_i is also an eigenvalue of

$$\Sigma_{11}^{-1} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$$

with eigenvector $\Sigma_{11}^{-\frac{1}{2}} \mathbf{e}_i$. *10*

Q10. Answer any ***two*** from the following :

(a) Consider the data matrix

$$X = \begin{pmatrix} -1 & 3 & -2 \\ 2 & 4 & 2 \\ 5 & 2 & 3 \end{pmatrix}.$$

(i) Calculate the matrix of deviations (residuals), $X - \mathbf{1}\bar{\mathbf{x}}'$. Is this matrix of full rank ? Explain.

(ii) Determine S and calculate the generalised sample variance $|S|$.

(iii) Using the result in (ii), calculate the total sample variance.

(iv) Obtain the distribution of generalised variance. *25*

(b) Determine the population principal components Y_1 and Y_2 for the covariance matrix

$$\Sigma = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}.$$

Also calculate the proportion of the total population variance explained by the first principal component. Convert the covariance matrix Σ into a correlation matrix ρ. Determine the principal components Y_1 and Y_2 from ρ and compute the total population variance explained by Y_1.

Compare the components calculated using Σ and ρ. Are they same ? Should they be ? *25*

(c) Test the null hypothesis $H_0 : \mu = \mu_0$ against an alternative hypothesis $H_1 : \mu \neq \mu_0$ when $\mathbf{x} \sim Np(\mu, \Sigma)$.

Let the data matrix for a random sample of size n = 3 from a bivariate normal population be

$$X = \begin{pmatrix} 4 & 1 \\ -1 & 3 \\ 3 & 5 \end{pmatrix}$$

(i) Evaluate T^2, for testing $H_0 : \mu' = (3, 2)$.

(ii) Specify the distribution of T^2 evaluated in (i).

(iii) Test H_0 at the $\alpha = 0{\cdot}05$ level of significance.

Write your conclusion.

($F_{2,\,1}\,(0{\cdot}05) = 200$, $F_{2,\,1}\,(0{\cdot}01) = 4999$, $F_{2,\,2}\,(0{\cdot}05) = 19{\cdot}00$

$F_{2,\,3}\,(0{\cdot}05) = 9{\cdot}55$, $F_{3,\,2}\,(0{\cdot}05) = 19{\cdot}16$, $F_{3,\,2}\,(0{\cdot}01) = 99{\cdot}17$) *25*

(d) Obtain a classification rule for classifying an individual $\mathbf{x}_0$ into one of the two populations Π_1 and Π_2 such that Expected Cost of Misclassification (ECM) is minimum.

Suppose that the density functions evaluated at a new observation $\mathbf{x}_0$ give $f_1(x_0) = 0{\cdot}35$ and $f_2(x_0) = 0{\cdot}45$, where $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are density functions associated with populations Π_1 and Π_2. If $c(1/2) = 80$ and $c(2/1) = 40$ units and it is known that about 30% of all objects belong to population Π_2, then classify $\mathbf{x}_0$ using minimum ECM rule. *25*

SECTION F
(Design and Analysis of Experiments)

Q11. (a) Discuss the Gauss-Markov linear model stating all assumptions underlying there. Also discuss how a general effect model and random effect model can be perceived as a particular case of Gauss-Markov model. *10*

(b) 32 plots are arranged in the form of a 4×8 rectangle. Suppose a 2^5 design with factors A, B, C, D and E at two-levels each is considered, where the effects ABC, CDE, ABDE are confounded in rows and AB, CD, ABCD, BDE, ADE, BCE, ACE with the columns. Suggest a confounded design arrangement for the above experiment. *10*

(c) A 2^2 factorial experiment was conducted to determine whether the type of glass and phosphor type affected the brightness of a television tube. The following table presents three response measures each of the current necessary (in micro-amps) to obtain a specified brightness level.

Measurement of current (in micro-amps)

Glass Type	Phosphor Type	
	A	B
1	280	300
	290	310
	285	295
2	230	260
	235	240
	240	235

Analyse the design and conclude at $\alpha = 5\%$.

[Given : $F_{[1, 8]} = 5 \cdot 32$, $F_{[2, 8]} = 4 \cdot 46$

$F_{[3, 8]} = 4 \cdot 07$, $F_{[4, 8]} = 3 \cdot 84$] *10*

(d) What are the advantages and disadvantages of split-plot design ? Also state the practical use of such a design. *10*

(e) Explain strip-plot design and its features. *10*

Q12. Answer any ***two*** from the following :

(a) An analysis of covariance (ANCOVA) for an RBD with 'b' blocks and 'v' treatments observed on an auxiliary variable 'x' related to the main variable 'y' is given as

$$y_{ij} = \mu + \beta_i + \tau_j + \gamma_{x_{ij}} + e_{ij},$$

$$i = 1, 2, \ldots, \phi,$$
$$j = 1, 2, \ldots, v$$

where $y_{ij}(x_{ij})$ is the yield of treatment j in block i and $e_{ij} \sim N(0, \sigma^2)$. The other parameters have usual meaning. Now investigate the following :

(i) Estimate the parameters : $(\mu, \beta_i, \tau_j, \gamma)$

(ii) Test $H_0 : \tau_1 = \tau_2 = \ldots = \tau_v$ against

H_1 : not all are equal, but at least one inequality is strict

(iii) Unbiased estimate of σ^2

(iv) ANCOVA table for RBD 25

(b) Consider the following model :

$$y_{ij} = \mu + \alpha_i + \beta_j + \Sigma_{ij},$$

$$i = 1, 2, \ldots, p,$$
$$j = 1, 2, \ldots, q$$

where α_i, β_j satisfy the condition $\sum_i \alpha_i = 0$, $\sum_j \beta_j = 0$, and Σ_{ij} is a random component with $E(\Sigma_{ij}) = 0$, $E(\sum_{ij}^2) = \sigma_e^2$. Prepare the ANOVA table for analysing the design and suggest the expected sum of squares due to all sources of variation. 25

(c) An online taxi company wants to consider four types of cars (Say : A, B, C, D) for its efficient management of realizing customer demands. This is tested using four car drivers on four different routes. The efficiency of cars is measured in terms of minutes to complete a particular distance. The layout and time consumed by each vehicle for each driver on each route are given below :

Time taken to cover a particular distance

Route	Drivers			
	1	2	3	4
1	18(C)	12(D)	16(A)	20(B)
2	26(D)	34(A)	25(B)	31(C)
3	15(B)	22(C)	10(D)	28(A)
4	30(A)	20(B)	15(C)	9(D)

Analyze the design and conclude for $\alpha = 0{\cdot}05$ level of significance.

[Given $\left.\begin{aligned} F_{(3, 6)} &= 4{\cdot}76 \\ F_{(4, 6)} &= 4{\cdot}53 \\ F_{(5, 6)} &= 4{\cdot}39 \end{aligned}\right\}$ for $\alpha = 0{\cdot}05$ 25

(d) In usual notations, obtain

(i) the efficiency of RBD with respect to CRD. 10

(ii) the efficiency of LSD with respect to RBD. 15

where

CRD : Completely Randomized Design

RBD : Randomized Block Design

LSD : Latin Square Design

SECTION G

(Computing with C and R)

Q13. (a) Why do we use Relational and Logical Operators in C ? List them with their syntaxes. *10*

(b) Write a C-program to compute median for grouped data. *10*

(c) Write a program in C to calculate Coefficient of Variation for a set of n observations. *10*

(d) Write R codes to fit a Binomial distribution to the following data : *10*

x	0	1	2	3	4	5	6	7
f	7	6	19	35	30	23	7	1

(e) Heights (in cm) of 8 randomly selected students from normal population with a standard deviation of 5 are given below :

168, 170, 185, 172, 169, 158, 170, 175

Write R-codes for performing Z-test for testing population mean height value of 140 cm at $\alpha = 0{\cdot}05$. *10*

Q14. Answer any ***two*** from the following :

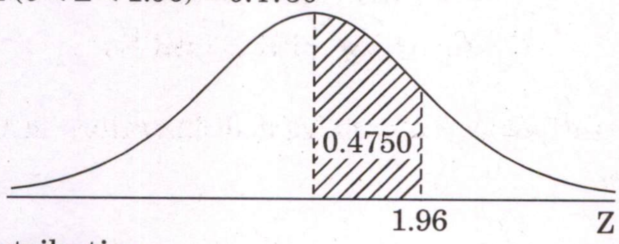
(a) Describe control statements in C with the help of an example in each case. *25*

(b) Write a C-program to obtain ANOVA table for a Randomized Block Design (RBD) having t-treatments in r-blocks. *25*

(c) Write a C-program to compute regression equation of Y on X for a given data set. *25*

(d) How do we read matrices in R ? Write R-codes for performing the following matrix operations : addition, multiplication, transposing, determinant and finding the inverse of a matrix. *25*

$P(0 < Z < 1.96) = 0.4750$



The Normal Distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000