I.E.S/ I.S.S. Examination, 2024

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO

T.B.C. : SGSE–D–STS **Test Booklet Series**

A

Serial No. 1003105

TEST BOOKLET

STATISTICS

Paper—II

Time Allowed : Two Hours **Maximum Marks : 200**

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. Consider the following statements :

I. The concentration of distribution of statistic $T(x)$ around its true value $f(\theta)$ is high when the mean square error is low but the converse is not true.

II. There exists no estimator for which the mean square error is least for all $\theta \in \Theta$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

2. Consider the following statements :

I. For an unbiased estimator, the variance of an estimator is same as its mean square error.

II. For a highly precise estimator, its bias is proportional to its mean square error.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

3. For a Minimum Variance Unbiased (MVU) estimator T_1 and another unbiased estimator T_2 which has an efficiency E_θ, which one of the following statements is correct?

(a) An unbiased linear combination of T_1 and T_2 is an MVU estimator.

(b) T_1 and T_2 are unique.

(c) The correlation coefficient between T_1 and T_2 is given by $(E_\theta)^{\frac{1}{2}}$.

(d) None of the above

4. Consider $X_i \sim N(\theta_1, \theta_2)$; $i = 1, 2, 3, \ldots, n$ such that θ_1 and θ_2 are both unknown. The C-R lower bound is

(a) attained for both the parameters

(b) attained for θ_1 but not for θ_2

(c) attained for θ_2 but not for θ_1

(d) not attained for either of the two parameters

5. Let

$$X_i \sim f_\theta(x) = \begin{cases} 1, & \text{if } \theta - \frac{1}{2} < x < \theta + \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

such that $\theta \in (-\infty, \infty)$; $i = 1, 2, ..., n$. Consider the following statements :

I. The distribution of $X_{(n)} - X_{(1)}$ is independent of θ.

II. $E_\theta[(X_{(n)} - X_{(1)}) - E(X_{(n)} - X_{(1)})] = 0$, for all θ.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

6. Let $U = U(X_1, X_2, X_3, ..., X_n)$ be an unbiased estimator of $\gamma(\theta)$ and T be a sufficient statistic for θ. Define $\phi(t) = E[U|T = t]$. Consider the following statements :

I. $\phi(t)$ is always independent of θ.

II. The variance of $\phi(t)$ is less than the variance of U.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

7. Consider the following statements :

I. The least attainable variance may be greater than the Cramer-Rao lower bound.

II. The Cramer-Rao inequality provides us with a means of judging whether or not a given unbiased estimator is also a minimum variance estimator.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

8. Consider the following statements :

I. It is possible to have a minimum variance unbiased estimator which is not minimum variance bound estimator.

II. Sufficient statistic should have a complete family of distributions to enable obtaining a minimum variance unbiased estimator from any unbiased estimator.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

9. Consider a series of Bernoulli trials with probability of success $p\,(0 < p < 1)$ for each trial, such that f is frequency of success in n trials. Consider the following statements :

I. $\frac{f}{n}\left(1-\frac{f}{n}\right)$ is a consistent estimator of $p(1-p)$.

II. $\frac{f}{n}$ is a consistent estimator of $\frac{1}{p}$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

10. Which of the following statements in respect of maximum likelihood estimates (MLEs) are correct?

I. MLEs are always functions of sufficient statistics.

II. Sometimes an MLE may not even exist.

III. MLEs are not necessarily unbiased.

IV. MLEs are unique.

Select the answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

11. Let

$$X_i \sim f(x) = \begin{cases} \frac{1}{N}; & x = 1, 2, \ldots, N \\ 0; & \text{otherwise} \end{cases};$$

$$i = 1, 2, 3, \ldots, n$$

The Maximum Likelihood Estimator (MLE) of N has which one of the following properties?

(a) Consistent, sufficient and complete

(b) Unbiased, sufficient and complete

(c) Unbiased and sufficient only

(d) Unbiased, complete and consistent

12. Let $X_i \sim N(\mu, \theta)$; $0 < \theta < \infty$; $i = 1, 2, 3, \ldots, n$ such that μ is known but θ is unknown. What is the sufficient statistic for θ?

(a) $\frac{1}{n-1}\left[\sum_{i=1}^{n}(X_i - \mu)\right]^2$

(b) $\frac{1}{n}\left[\sum_{i=1}^{n}(X_i - \mu)\right]^2$

(c) $\frac{1}{n}\sum_{i=1}^{n-1}(X_i - \mu)^2$

(d) $\sum_{i=1}^{n}(X_i - \mu)^2$

13. Which one of the following statements is correct in respect of a parametric function?

(a) A parametric function is said to be estimable if its unbiased estimator exists.

(b) A parametric function is said to be estimable if its minimum variance unbiased estimator exists.

(c) A parametric function is said to be estimable if its mean square error is minimum.

(d) None of the above

14. Consider the following statements :

I. $X_i \sim N(\theta, \sigma^2)$; σ^2 known and θ unknown; $i = 1, 2, 3, \ldots, n$.

II. $X_i \sim$ Bernoulli (θ); θ unknown; $i = 1, 2, 3, \ldots, n$.

The variance of sample mean coincides with the Cramer-Rao lower bound for every θ, for

(a) I only

(b) II only

(c) both I and II

(d) neither I nor II

15. Consider the following statements :

I. Uniformly most accurate lower confidence bound has the smallest probability of false coverage.

II. Confidence level has the smallest probability of true coverage.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

16. Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a Poisson distribution with mean $\theta > 0$. Let $T = \sum_{i=1}^{n} x_i$. Then UMVUE of $e^{-2\theta}\theta^3$ is

(a) $\dfrac{(n-2)^{T-3}}{n^T}$

(b) $\dfrac{(n-2)^T\, T(T-1)(T-2)}{T!}$

(c) $\dfrac{T(T-1)(T-2)}{n^T}$

(d) $\dfrac{(n-2)^{T-3}\, T(T-1)(T-2)}{n^T}$

17. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from $N(0, \sigma^2)$, $\sigma > 0$. Which one of the following hypotheses has the critical region $\sum_{i=1}^{n} x_i^2 \geq 2\chi^2_{(n,\,\alpha)}$ as the UMP critical region of level α, where $P(\chi^2_{(n)} > \chi^2_{(n,\,\alpha)}) = \alpha$?

(a) $H_0 : \sigma^2 = 2$ versus $H_1 : \sigma^2 > 2$

(b) $H_0 : \sigma^2 = 1$ versus $H_1 : \sigma^2 > 1$

(c) $H_0 : \sigma^2 = 4$ versus $H_1 : \sigma^2 < 4$

(d) $H_0 : \sigma^2 = 2$ versus $H_1 : \sigma^2 < 2$

18. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from the distribution having pdf $f(x, \mu) = 3e^{-3(x-\mu)}$; $x > \mu$, $\mu \in R$. If $X_{(1)} = \min\{X_1, X_2, X_3, ..., X_n\}$, then which one of the following is correct?

(a) $X_{(1)}$ is MVUE for μ.

(b) $X_{(1)}$ is unbiased but not MVUE for μ.

(c) $X_{(1)}$ is biased and consistent for μ.

(d) $X_{(1)}$ is neither biased nor consistent for μ.

19. Let $X_1, X_2, X_3, ..., X_n$ be a sequence of iid observations from an exponential distribution with mean $\dfrac{1}{\theta}$, $\theta > 0$. To test $H_0 : \theta = 1$ versus $H_1 : \theta = 2$, SPRT is used. What is the probability that the procedure has no decision at the first stage when $\alpha = \beta = 0{\cdot}1$ and H_0 is true?

(a) $\dfrac{1}{18}$ (b) $\dfrac{2}{9}$

(c) $\dfrac{8}{9}$ (d) $\dfrac{17}{18}$

20. If $x_1, x_2, x_3, ..., x_n$ is a random sample from $N(0, \theta)$, then which of the statements given below is/are correct?

I. $T_1 = x_1$ is complete statistic for θ.

II. $T_2 = x_1^2$ is complete statistic for θ.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

21. Let $x_1, x_2, x_3, ..., x_{16}$ be a random sample from $N(\mu, 1)$ distribution, $\mu \in R$. Consider two tests $\varphi_1(x)$ and $\varphi_2(x)$ to test $H_0 : \mu = 3$ versus $H_1 : \mu = 4$:

$$\varphi_1(x) = \begin{cases} 1, & x_1 \geq 5 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and}$$

$$\varphi_2(x) = \begin{cases} 1, & \bar{x} \geq 3{\cdot}5 \\ 0, & \text{elsewhere} \end{cases}$$

If α_i and β_i; $i = 1, 2$ are size and type II error of the two tests respectively, then which one of the following is correct?

(a) $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$

(b) $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$

(c) $\alpha_1 = \alpha_2$ and $\beta_1 > \beta_2$

(d) $\alpha_1 < \alpha_2$ and $\beta_1 = \beta_2$

22. Let $x_1, x_2, x_3, ..., x_n$ be a random sample from the distribution having pdf $f(x, \theta) = \frac{1}{\theta} e^{-\frac{(x-\theta)}{\theta}}$; $x \geq \theta$, $\theta > 0$. The maximum likelihood estimator of $ln\,\theta$ is

(a) $ln\left[\frac{\sum_{i=1}^{n} x_i}{n} + x_{(1)}\right]$

(b) $ln\left[\frac{\sum_{i=1}^{n} x_{(i)}}{n} - x_{(1)}\right]$

(c) $ln[x_{(1)}]$

(d) $ln\left[\sum_{i=1}^{n} x_{(i)} - n\right]$

23. Let $x_1, x_2, x_3, ..., x_n$ be a random sample from the gamma distribution having pdf

$$f(x; \theta, \mu) = \frac{1}{\Gamma(\theta)\mu^{\theta}} e^{-\frac{x}{\mu}} x^{\theta-1}; \quad x > 0, \ \theta > 0$$

and $\mu > 0$ is known constant. What is the sufficient statistic for θ?

(a) $\sum_{i=1}^{n} x_i$

(b) $x_{(1)} = \min\{x_1, x_2, x_3, ..., x_n\}$

(c) $x_{(n)} = \max\{x_1, x_2, x_3, ..., x_n\}$

(d) $\prod_{i=1}^{n} x_i$

24. Let X be a random sample with probability mass function under H_0 and H_1 given by

X	1	2	3	4	5	6	
$f_0(x)$	0·01	0·01	0·01	0·01	0·01	0·95	Under H_0
$f_1(x)$	0·05	0·04	0·03	0·02	0·01	0·85	Under H_1

The Neyman-Pearson MP size 0·03 test rejects H_0 if $x \leq 3$. What is P(type II error) equal to?

(a) 0·09

(b) 0·12

(c) 0·85

(d) 0·88

25. Consider the following statements :

Statement–I :

Every Borel measurable mapping f of $R_n \to [0, 1]$ is called a test function.

Statement–II :

Whenever f is the indicator function of some Borel subset A of R_n, then A is called the acceptance region of the test f.

Which one of the following is correct in respect of the above statements?

(a) Both Statement–I and Statement–II are correct and Statement–II explains Statement–I

(b) Both Statement–I and Statement–II are correct but Statement–II does not explain Statement–I

(c) Statement–I is correct but Statement–II is incorrect

(d) Statement–I is incorrect but Statement–II is correct

26. Consider the following statements :

I. Smaller the P-value, more extreme is the outcome and stronger is the evidence in favour of H_0, the null hypothesis.

II. The Neyman-Pearson most powerful test is not always a function of sufficient statistic, provided the latter exists for a family of distributions.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

27. Let $X_1, X_2, X_3, ..., X_n$ be independent random variables each $N(\mu, \sigma^2)$, with μ and σ^2 both unknown. Then UMVUE of μ and σ^2 are respectively

(a) $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} X_i^2$

(b) $\sum_{i=1}^{n} X_i^2$ and $\sum_{i=1}^{n} X_i$

(c) $\bar{X}$ and $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

(d) $\bar{X}$ and $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

28. The Neyman-Pearson lemma gives a general method for finding a most powerful test for testing

(a) simple null hypothesis against composite alternative hypothesis

(b) composite null hypothesis against simple alternative hypothesis

(c) simple null hypothesis against simple alternative hypothesis

(d) composite null hypothesis against composite alternative hypothesis

29. Consider the following statements regarding likelihood ratio tests :

I. For a given size α of a test, the non-randomized Neyman-Pearson and likelihood ratio tests of a simple hypothesis against a simple alternative hypothesis are equivalent.

II. The likelihood ratio λ always lies between 0 and 1.

III. For testing a null hypothesis against any alternative hypothesis, the likelihood ratio test is a function of every sufficient statistic for θ.

IV. If λ is the likelihood ratio, then the asymptotic distribution of $-2\ln\lambda$ is normal with mean 0 and variance 1.

Which of the statements given above are correct?

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

30. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from the double exponential distribution having pdf $f(x, \theta) = \frac{1}{2}e^{-|x-\theta|}$; $\theta \in R$, $x \in R$. Which one of the following statements is correct?

(a) No alone sufficient statistic T exists for θ, other than the sample itself.

(b) Sample median is unbiased estimator of θ.

(c) $X_{(n)}$ is sufficient as well as MLE for θ.

(d) Sample mean is MLE and consistent for θ.

31. If T_1 and T_2 are equally efficient estimators of $g(\theta)$ with efficiency 0·8, then the correlation coefficient between them lies in the interval

(a) (0·8, 1)

(b) (–1, 0·8)

(c) (–0·64, 0·64)

(d) (0·6, 1)

32. Let $x_1, x_2, x_3, ..., x_{10}$ be a random sample from a uniform $U(-\theta, \theta)$, $\theta > 0$ distribution. It is given that the values of the largest and smallest observations in the sample are 7 and –12 respectively. Then MLE of θ is

(a) –12

(b) $-\frac{5}{2}$

(c) 7

(d) 12

33. Let {0, 4, 2, 3} be a random sample from a Poisson distribution with mean $\theta > 0$. Then the asymptotic confidence interval with confidence coefficient 0·95 for mean θ is

(a) (0·78, 3·72)

(b) (0·78, 4·72)

(c) (2·25, 3·72)

(d) (2·25, 4·72)

34. Let $\{x_1, x_2, x_3\}$ be a random sample of size 3 from a Poisson distribution with mean $\lambda > 0$. To test $H_0 : \lambda = \frac{1}{8}$ versus $H_1 : \lambda = 1$, reject H_0 when $\sum_{i=1}^{3} x_i > 1$. The power of the test is approximately (given $e^{-3} = 0{\cdot}04978$)

(a) 0·80

(b) 0·90

(c) 0·95

(d) 0·99

35. Consider the following statements :

I. Bayes estimator with a constant risk is a minimax estimator.

II. If a minimax estimator of a parametric function is unique, then it must be admissible.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

36. A biased coin with probability of head p is tossed m times independently. It is known that $p \in \{1/4, 3/4\}$ and $m \in \{3, 5\}$. If 3 heads are observed in these m tosses, then which one of the following is correct?

(a) (5, 3/4) is MLE of m and p

(b) (5, 1/4) is MLE of m and p

(c) (3, 3/4) is MLE of m and p

(d) MLE of m and p is not unique

37. Consider a random sample $\{x_1, x_2, x_3, x_4\}$ from $N(\mu, \sigma^2)$ distribution. Define $\bar{x} = \frac{\sum_{i=1}^{4} x_i}{4}$ and $S^2 = \frac{\sum_{i=1}^{4}(x_i - \bar{x})^2}{3}$. The LRT of size $\alpha = 0{\cdot}05$ rejects $H_0 : \mu = 0$ versus $H_1 : \mu \neq 0$ if and only if $\frac{|\bar{x}|}{S} > k$. The value of k, when $P(X > t_{(n, \alpha)}) = \alpha$, is

(a) $\frac{1}{2} t_{(3,\ 0{\cdot}05)}$ *(b)* $\frac{1}{2} t_{(3,\ 0{\cdot}025)}$

(c) $2t_{(4,\ 0{\cdot}025)}$ *(d)* $2t_{(4,\ 0{\cdot}05)}$

38. Let $x_1, x_2, x_3, \ldots, x_n$ denote a random sample from a distribution with pdf $f(x; \theta) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$, where $\theta > 0$. The sufficient statistic for θ is

(a) $\sum_{i=1}^{n} x_i$

(b) $\prod_{i=1}^{n} x_i$

(c) $[\min x_i, \max x_i]$

(d) $\max x_i - 1$

39. Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from a distribution with probability density function $f(x; \theta) = \frac{2x}{\theta} e^{-\frac{x^2}{\theta}}$; $x > 0$, $\theta > 0$. Which of the following statements are correct in respect of parameter θ of the distribution?

I. The maximum likelihood estimate of θ is given by $\hat{\theta}_{MLE} = \frac{1}{n}\sum_{i=1}^{n} x_i^2$.

II. $\hat{\theta}_{MLE}$ is an unbiased estimator of θ.

III. $\hat{\theta}_{MLE}$ is sufficient estimator of θ.

Select the answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

40. Let $x_1, x_2, x_3, \ldots, x_m$ denote a random sample from a binomial $B(n, p)$ distribution. Define a statistic $u = \begin{cases} \frac{1}{2n}, & \text{if } x_1 + x_2 = 1 \\ 0, & \text{otherwise} \end{cases}$. Which of the following statements is/are correct?

I. u is an unbiased estimator of $\frac{p(1-p)}{n}$.

II. u is a uniformly minimum variance unbiased estimator of p.

III. $\sum_{i=1}^{m} x_i$ is a complete sufficient statistic for p.

Select the answer using the code given below.

(a) I and II

(b) III only

(c) II and III

(d) I and III

Consider the following for the next **two (2)** items that follow :

Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample of size n from a normal distribution $N(\mu, 1)$. $w_0 = \{(x_1, x_2, x_3, ..., x_n); \bar{X} > \theta\}$ is critical region of size α for testing $H_0 : \mu = 0$ against $H_1 : \mu = 4$.

41. What is the value of θ?

(a) Z_α (b) $\frac{Z_\alpha}{n}$

(c) $\frac{Z_\alpha}{\sqrt{n}}$ (d) $\sqrt{n} Z_\alpha$

42. What is the power of the test?

(a) $\Phi\left(\frac{4}{\sqrt{n}} - Z_\alpha\right)$

(b) $\Phi(4\sqrt{n} - Z_\alpha)$

(c) $\Phi\left(-\frac{Z_\alpha}{2}\right)$

(d) $\Phi(4Z_\alpha - \sqrt{n})$

Consider the following for the next **two (2)** items that follow :

To test mean of the Poisson distribution $H_0 : \lambda = 1$ versus $H_1 : \lambda = 2$, based on the single observed value from the Poisson distribution, the following procedure is considered :

"Reject H_0 with probability one when the observed value $(x) > 1$. If $x = 1$ is observed, then toss an unbiased coin and reject the null hypothesis if head comes up."

43. What is type I error in the experiment?

(a) $1 - e^{-1}$ (b) $1 - 0.5e^{-1}$

(c) $1 - 1.5e^{-1}$ (d) $1 - 2e^{-1}$

44. What is type II error in the experiment?

(a) $1 - e^{-2}$

(b) $1 - 0.5e^{-2}$

(c) $1 - 2e^{-2}$

(d) $2e^{-2}$

Consider the following for the next **two (2)** items that follow :

Assume a sample of continuous random variables $X_1, X_2, X_3, ..., X_n$ such that $E(X_i) = \mu$, $\text{Var}(X_i) = \sigma^2 > 0$; $i = 1, 2, 3, ..., n$.

Define $\hat{\mu}_{1,n} = X_n$ and $\hat{\mu}_{2,n} = \frac{1}{n+1}\sum_{i=1}^{n} X_i$.

45. Consider the following statements :

I. $\hat{\mu}_{1,n}$ is an unbiased estimator of μ.

II. $\hat{\mu}_{2,n}$ is an unbiased estimator of μ.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

46. Consider the following statements :

I. $\hat{\mu}_{1,n}$ is a consistent estimator of μ.

II. $\hat{\mu}_{2,n}$ is a consistent estimator of μ.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

Consider the following for the next **two (2)** items that follow :

Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from a distribution with pdf $f(x|\alpha, \beta) = \frac{e^{-\frac{x}{\beta}} x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}$; $x > 0$, $\alpha > 1$, $\beta > 0$.

47. What is the estimator of α obtained by method of moments?

(a) $\frac{n\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$

(b) $\frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$

(c) $\frac{\bar{X}}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$

(d) $\frac{n\bar{X}}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$

48. What is the estimator of β obtained by method of moments?

(a) $\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n\bar{X}}$

(b) $\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\bar{X}}$

(c) $\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\bar{X}^2}$

(d) $\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n\bar{X}^2}$

Consider the following for the next **two (2)** items that follow :

Let $(X_1, X_2, X_3, ..., X_n)$ be independently and identically distributed random sample having pdf $f(x) = \theta x^{\theta-1}$; $0 \le x \le 1$, $0 < \theta < \infty$.

49. What is $E(X)$ equal to?

(a) $\frac{\theta}{\theta+1}$

(b) $\frac{\theta}{\theta-1}$

(c) $\frac{2\theta}{\theta+1}$

(d) $\frac{2\theta}{\theta-1}$

50. What is the moment estimator of θ?

(a) $\frac{\sum_{i=1}^{n} X_i}{n}$

(b) $\frac{\sum_{i=1}^{n} X_i}{n - \sum_{i=1}^{n} X_i}$

(c) $\frac{\sum_{i=1}^{n} X_i}{1 + \sum_{i=1}^{n} X_i}$

(d) $\frac{\sum_{i=1}^{n} X_i}{n + \sum_{i=1}^{n} X_i}$

51. Consider the following statements in respect of a symmetric matrix A :

I. Generalized inverse of A is not necessarily symmetric.

II. Symmetric generalized inverse of A can always be found.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

52. Let A be an $n \times p$ matrix of rank r and let A^- denote the generalized inverse of A. Consider the following statements :

I. Trace (AA^-) = Trace (A^-A)

II. Trace $(AA^-) \neq r$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

53. If the system of equations $Ax = c$ has at least one solution, then which of the following statements is/are correct?

I. Rank (A) = Rank (A, c)

II. $AA^-c = c$, where A^- is the generalized inverse of A.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

54. If $A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}$ and A^- denotes the generalized inverse of A, then consider the following statements :

I. Rank $(A) = 2$

II. Trace $(A^-A) = 2$

III. Trace (AA^-) = Trace (A^-A)

IV. Trace $(A) = 6$

How many of the statements given above are correct?

(a) Only one statement

(b) Only two statements

(c) Only three statements

(d) All four statements

55. Let A^- be any generalized inverse of A and $A^-A = H$. Consider the following statements :

I. $AH = A$ and $HA^- = A^-$

II. Trace (H) = Rank (H)

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

56. For the matrices

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

consider the following statements :

I. $ABA = A$

II. BA is an idempotent matrix.

III. Rank $(A) \neq$ Rank (BA)

IV. Rank (BA) = Trace (BA)

How many of the statements given above are correct?

(a) Only one statement

(b) Only two statements

(c) Only three statements

(d) All four statements

57. Which of the following statements are correct regarding ANOVA technique?

I. It is used to test the equality of several population means.

II. It is used to test the equality of several population variances.

III. Its origin lies in agricultural experiments.

Select the answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

58. In ANOVA for one-way classified data, which of the following statements are correct?

I. The value of various sum of squares is independent of change of origin.

II. The value of various sum of squares is independent of change of scale.

III. The value of variance ratio is independent of change of origin.

IV. The value of variance ratio is independent of change of scale.

Select the answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

59. Consider the random effect model for a two-way classified data with m observations per cell. Which of the following statements is/are correct?

I. Mean sum of squares due to error is used as error variance for computing the test statistics for $H_{0(A\times B)} : \sigma^2_{ab} = 0$.

II. Mean sum of squares due to error is used as error variance for computing the test statistics for $H_{0A} : \sigma^2_a = 0$.

III. Mean sum of squares due to error is used as error variance for computing the test statistics for $H_{0B} : \sigma^2_b = 0$.

Select the answer using the code given below.

(a) I

(b) II only

(c) III only

(d) II and III

60. For the validity of F-test in ANOVA, which of the following are assumed?

I. The observations are independent.

II. Parent population from which observations are taken is normally distributed.

III. The variances of the population from which observations are taken may differ from one another.

IV. Different effects are additive in nature.

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

61. The Registrar General of India is responsible for which of the following activities?

I. Civil Registration System

II. Socio-economic Indicators

III. Housing and Population Census

IV. Mother Tongue Survey

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

62. Which of the following resources fall under the category of Natural Capital?

I. Computer and life-saving devices

II. Energy

III. Timber

IV. Agricultural land

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

63. Data for the handicraft/handloom establishments were collected for the first time during which of the following Economic Censuses?

(a) Third Economic Census

(b) Fourth Economic Census

(c) Fifth Economic Census

(d) Sixth Economic Census

64. Who among the following is the Secretary of the National Statistical Commission?

(a) The Director General, Central Statistics Office

(b) The Secretary, Ministry of Statistics and Programme Implementation

(c) The Chief Statistician of India

(d) The Registrar General and Census Commissioner of India

65. Consider the following :

I. Crops

II. Livestock

III. Forestry and logging

IV. Fishing and aquaculture

V. Mining and quarrying

In the estimates of GDP, how many of the above sectors/subsectors comprise the 'primary sector'?

(a) Only two

(b) Only three

(c) Only four

(d) All five

66. Consider the following regarding a 'longitudinal panel survey' :

I. A variable set of respondents surveyed once

II. A random set of respondents surveyed twice

III. A fixed set of respondents surveyed repeatedly over time

IV. A new set of respondents surveyed whenever required

How many of the above is/are correct?

(a) Only one

(b) Only two

(c) Only three

(d) All four

67. Which of the following statements regarding official statistics are correct?

I. Official statistics that fulfill the test of practical utility are to be compiled.

II. Official statistics may be drawn from all types of sources (statistical surveys or administrative records).

III. The laws, regulations and measures under which the statistical systems operate must be made public.

Select the answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

68. Statistics regarding which of the following aspects and matters connected therewith may be collected under the Collection of Statistics Act, 2008?

I. Economic and demographic

II. Social and scientific

III. Environmental

Select the correct answer using the code given below.

(a) I and II only

(b) II and III only

(c) I and III only

(d) I, II and III

69. Which of the following statements regarding the Consumer Price Index-Industrial Workers (CPI-IW) are correct?

I. CPI-IW is computed by the Labour Bureau.

II. CPI-IW is used as a tool for measuring inflation.

III. CPI-IW is utilized for regulations of wages and dearness allowance for workers and employees in the country.

IV. CPI-IW is computed for male, female, rural areas and urban areas.

Select the answer using the code given below.

(a) I and III only

(b) II and IV only

(c) I, II and III only

(d) I, II, III and IV

70. Consider the following Censuses :

I. Economic Census

II. Minor Irrigation Census

III. Labour Census

IV. Livestock Census

How many of the above Censuses are conducted by the Government of India?

(a) Only one

(b) Only two

(c) Only three

(d) All four

71. Consider the following statements with regard to the National Statistical Commission (NSC) :

I. NSC was set up by the Government of India through a resolution in the year 2005.

II. NSC is a constitutional body.

III. NSC has four Members besides a Chairperson.

Which of the statements given above is/are correct?

(a) I only

(b) I and III only

(c) II and III only

(d) I, II and III

72. Which of the following are meant to provide legal support for collection of official statistics in India?

I. The Government of India (Allocation of Business) Rules, 1961, as amended from time to time

II. The Census Act, 1948

III. The Representation of the People Act, 1951

IV. The Collection of Statistics Act, 2008

Select the correct answer using the code given below.

(a) I, II and III

(b) I, II and IV

(c) I, III and IV

(d) II, III and IV

73. Which of the following features differentiates/differentiate GDP from Gross National Income?

I. Monetary

II. Geographic

III. Strategic

IV. Political

Select the correct answer using the code given below.

(a) I

(b) II only

(c) II and III

(d) III and IV

74. Which of the following are computed as per the prevalent methodology of estimation of national income?

I. State Domestic Product

II. District Domestic Product

III. Urban Domestic Product

IV. Rural Domestic Product

Select the correct answer using the code given below.

(a) I and II

(b) II and III

(c) III and IV

(d) I and IV

75. Which of the following are accepted methods of data collection?

I. Census and surveys

II. Administrative records

III. Management information systems

IV. Remote sensing

Select the correct answer using the code given below.

(a) I, II and III only

(b) II, III and IV only

(c) I and IV only

(d) I, II, III and IV

76. In the construction of a price index number, which of the following factors has/have to be considered?

I. The selection of commodities must be representative.

II. Comparison across different places whether national or international must be possible.

III. It must be applicable to each individual within the group for which the index is being prepared.

Select the correct answer using the code given below.

(a) I only

(b) I and II

(c) I and III

(d) II and III

77. Consider the following statements :

I. Index numbers measure changes in the magnitude of a group of related variables.

II. Index numbers measure the extent to which value in a distribution differs from the average value.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

78. The major source of data for Index of Industrial Production (IIP) is from the

(a) Confederation of Indian Industry

(b) Department for Promotion of Industry and Internal Trade

(c) National Sample Survey

(d) All India Association of Industries

79. Consider the following statements :

I. The Gross Enrolment Ratio (GER) of India indicates that, in principle, India is able to accommodate all of its primary school-age population.

II. The Gender Parity Index shows a decreasing trend at Primary and Higher Education level over the years from 2016–17 to 2021–22.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

80. Which one of the following statements is correct if the money wage of an individual is ₹ 8,000 and the Consumer Price Index (2012 = 100) is 200 in January 2023?

(a) The real wage of the individual will be ₹ 4,000 in January 2023.

(b) The equivalent of a rupee in January 2023 is 40 paisa in 2012.

(c) If the individual was getting ₹ 6,000 in 2012, she/he is able to maintain the same standard of living in 2023.

(d) The purchasing power of money remains the same in both the years 2012 and 2023.

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