IES/ISS EXAM,2017

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**T.B.C. : STT–D–DFHH**

**Test Booklet Series**

**Serial No.** 0008465

# A

# TEST BOOKLET

## STATISTICS

PAPER—II

**Time Allowed : Two Hours**

**Maximum Marks : 200**

## INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write *anything else* on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. The following ANOVA table is obtained for a two-variable linear regression :

| *Source of variation* | *Degrees of freedom* | *Sum of squares* |
|---|---|---|
| Regression | 1 | 122·5 |
| Residual | 3 | 77·5 |
| *Total* | 4 | 200·0 |

The correlation between the regressor and the regressand is

(a) 0·3875

(b) 0·6125

(c) 0·6225

(d) 0·7826

2. In a 2-way ANOVA with 6 rows, 5 columns and 3 observations per cell, what are the degrees of freedom for the sum of squares for interaction and error respectively?

(a) 20 and 60

(b) 30 and 50

(c) 20 and 40

(d) 15 and 65

3. If $Y_1$, $Y_2$, $Y_3$ and $Y_4$ are independent with

$$E(Y_1) = E(Y_2) = \mu_1 + \mu_2$$

$$E(Y_3) = E(Y_4) = \mu_1 + \mu_3$$

$$\text{Var}(Y_i) = \sigma^2;\ i = 1, 2, 3, 4$$

the condition of estimability of the parametric function $l_1\mu_1 + l_2\mu_2 + l_3\mu_3$ is

(a) $l_2 = l_3$

(b) $l_2 = l_1 + l_3$

(c) $l_3 = l_1 + l_2$

(d) $l_1 = l_2 + l_3$

4. Let $y_i$ follow

$$N(\alpha + \beta x_i, \sigma^2);\ i = 1, 2, 3, \ldots, n$$

Let $(\tilde{\alpha}, \tilde{\beta})$ and $(\hat{\alpha}, \hat{\beta})$ be the least squares and maximum likelihood estimates of $(\alpha, \beta)$ respectively. Then which one of the following is correct?

(a) $\tilde{\alpha} \neq \hat{\alpha},\ \tilde{\beta} \neq \hat{\beta}$

(b) $\tilde{\alpha} = \hat{\alpha},\ \tilde{\beta} \neq \hat{\beta}$

(c) $\tilde{\alpha} = \hat{\alpha},\ \tilde{\beta} = \hat{\beta}$

(d) $\tilde{\alpha} = \hat{\alpha},\ \tilde{\beta} = \hat{\beta}$ only when $\sigma^2$ is known

5. In a multiple linear regression model with 5 regressors and 25 observations, the sum of squares due to regressors was 360 and the sum of squares due to errors was 128. For this model, consider the following statements :

1. Almost 74% of the total variation in the model is explained by the regressors.
2. The estimated value of the error variance of the model is 3·2.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

6. In the following, the Gauss-Markov model is stated as $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ and $\mathbf{X}^{-}$ denotes the g-inverse of the matrix $\mathbf{X}$ :

1. $\mathbf{X}\mathbf{X}^{-}\mathbf{X} = \mathbf{X}$
2. If $\mathbf{X}$ is non-singular, then $\mathbf{X}^{-} = \mathbf{X}^{-1}$
3. $\mathbf{X}^{-}$ is needed when the columns of $\mathbf{X}$ are linearly dependent

Which of the above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* 1 and 2 only

*(d)* 1, 2 and 3

7. In the Gauss-Markov model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, which of the following assumptions are made?

1. $\mathbf{X}$ is a matrix of full rank
2. $E(\boldsymbol{\varepsilon}) = \mathbf{0}$
3. $\text{Var}(\boldsymbol{\varepsilon}) = \boldsymbol{\sigma}^2\mathbf{I}$

Select the correct answer using the code given below.

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

8. An experiment was conducted to study the variation in fuel consumption of diesel vehicles. For this purpose, different types of fuel injectors (*A*) and different qualities of fuel (*B*) were assigned randomly to the same type of diesel vehicles. The analysis of variance table of the random effect model is given below :

ANOVA Table

| *Source* | *Degrees of freedom* | *Sum of squares* |
|---|---|---|
| *A* | 4 | 48 |
| *B* | 3 | 36 |
| $A \times B$ | $x$ | 96 |
| *Error* | $y$ | $z$ |
| *Total* | 59 | 380 |

The estimate of component of variance due to interaction ($\hat{\sigma}^2_{AB}$) is

*(a)* 1

*(b)* 1·6

*(c)* 5

*(d)* 8

9. Consider the following two models with $E(\boldsymbol{\varepsilon}) = \mathbf{0}$ and $\text{Var}(\boldsymbol{\varepsilon}) = \sigma^2\mathbf{I}$ :

Model $A$ : $\mathbf{Y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}$

Model $B$ : $\mathbf{Y} = \mathbf{X}_1'\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}$

Which one of the following is correct?

(a) $R_A^2 = R_B^2$

(b) $R_A^2 \le R_B^2$

(c) $R_A^2 \ge R_B^2$

(d) $R_A^2 \ne R_B^2$

10. Consider a model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, where $\mathbf{Y}$ is $n \times 1$, $\mathbf{X}$ is $n \times p$ and $\boldsymbol{\beta}$ is $p \times 1$. Let $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-}\mathbf{X}'$. Which of the following are correct?

1. $(\mathbf{I}_n - \mathbf{H})$ is symmetric
2. $(\mathbf{I}_n - \mathbf{H})$ is idempotent
3. $(\mathbf{I}_n - \mathbf{H})\mathbf{X} = \mathbf{0}$
4. $(\mathbf{I}_n - \mathbf{H})\mathbf{X} \ne \mathbf{0}$

Select the correct answer using the code given below.

(a) 1, 2 and 4

(b) 2 and 3 only

(c) 1, 2 and 3

(d) 1 and 3 only

11. Let $n$ independent random variables $y_1, y_2, \ldots, y_n$ be normally distributed with constant and unknown variance $\sigma^2$ and

$$E(y_i) = \sum_{j=0}^{p} \beta_j x_{ij}\,; \quad x_{i0} = 1$$

for all $i = 1, 2, 3, \ldots, n$

The distribution of the test statistic for testing the hypothesis

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_p = 0$$

is

(a) $t_{(n-p)}$

(b) $F_{(p,\, n-p-1)}$

(c) $F_{(p-1,\, n-p)}$

(d) $t_{(p-1)}$

12. The following information is given for a two-variable regression model that was run on 22 observations :

$$\Sigma(y_i - \bar{y})^2 = 500,\ \Sigma(x_i - \bar{x})^2 = 125,\ r_{xy} = 0{\cdot}6$$

The estimate of error variance will be given by

(a) 1·20

(b) 1·44

(c) 9·00

(d) 16·00

13. The main advantage of having more than one observation per cell in two-way ANOVA is

1. increased degrees of freedom for error SS
2. testing significance of interaction effect

Which of the above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

14. For a one-way ANOVA in $k$ classes, each with $n$ observations, consider the following :

1. Least significant difference

$$= \sqrt{\frac{2 \times \text{MSE}}{n}} \times t_{(n-1)k,\ \alpha/2}$$

2. Scheffe's critical difference

$$= \sqrt{\frac{2 \times \text{MSE}}{n} \times (k-1) \times F_{k-1,\,(n-1)k,\,1-\alpha}}$$

Which of the above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

15. Consider the model $E(y_1) = 2\beta_1 + \beta_2$, $E(y_2) = \beta_1 - \beta_2$ and $E(y_3) = \beta_1 + \alpha\beta_2$ with $V(y_i) = \sigma^2$ for all $i = 1, 2, 3$ and $\text{Cov}(y_i, y_j) = 0$ for all $i \neq j$. The value of $\alpha$ so that the best linear unbiased estimators of $\beta_1$ and $\beta_2$ are uncorrelated is

*(a)* 0

*(b)* 2

*(c)* $-2$

*(d)* $-1$

16. To estimate $\boldsymbol{\beta}$ in general linear model $\mathbf{Y}_{n\times1} = \mathbf{X}_{n\times k}\,\boldsymbol{\beta}_{k\times1} + \boldsymbol{\varepsilon}_{n\times1}$, which one of the following assumptions is essential?

*(a)* $E(\boldsymbol{\varepsilon}) = \mathbf{0}$

*(b)* $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \boldsymbol{\sigma}^2\mathbf{I}_n$

*(c)* $\rho(\mathbf{X}) = k$

*(d)* $\boldsymbol{\varepsilon}$ follows $N(\mathbf{0}, \boldsymbol{\sigma}^2\mathbf{I}_n)$

17. Test for the significance of complete regression is equivalent to

*(a)* test for the significance of individual regression coefficients

*(b)* test for the equality between two regression coefficients

*(c)* test for the significance of multiple determination

*(d)* test for the regression coefficient equal to some given value

**18.** The included explanatory variable in the linear model is irrelevant if

*(a)* the estimate of the regression coefficient of explanatory variable is less than one

*(b)* the estimate of the regression coefficient of explanatory variable is greater than one

*(c)* the calculated *t*-statistic of the regression coefficient of explanatory variable is greater than one

*(d)* the calculated *t*-statistic of the regression coefficient of explanatory variable is less than one

**19.** The estimate of error variance in two-variable linear model is

*(a)* $\frac{\mathbf{e}'\mathbf{e}}{n-2}$

*(b)* $\frac{\mathbf{e}\mathbf{e}'}{n-2}$

*(c)* $\frac{\mathbf{e}'\mathbf{e}}{2-n}$

*(d)* $\frac{\mathbf{e}\mathbf{e}'}{2-n}$

**20.** For the linear hypothesis

$$H_0 : \mathbf{R}_{q\times k}\,\boldsymbol{\beta}_{k\times 1} = \mathbf{r}_{q\times 1}$$

which one of the following represents coefficients of linear restrictions on parameters involved in the model?

*(a)* $\mathbf{R}$

*(b)* $\boldsymbol{\beta}$

*(c)* $\mathbf{r}$

*(d)* $k$

**21.** The following table gives the observed frequencies for the four blood groups along with expected frequencies according to a model :

| *Blood group* | O | A | B | AB |
|---|---|---|---|---|
| *Observed* | 17 | 22 | 15 | 6 |
| *Expected* | 14 | 26 | 12 | 8 |

What is the corrected value of chi-square statistic after applying Yates' correction for continuity?

*(a)* 2·4874

*(b)* 2·2378

*(c)* 2·5272

*(d)* 2·7590

**22.** Which of the following are correct for a test of significance?

1. Size = 1 − *P* (Type–I error)
2. Power = 1 − *P* (Type–II error)
3. Power = *P* (Type–II error)
4. Size = *P* (Type–I error)

Select the correct answer using the code given below.

*(a)* 1 and 2

*(b)* 3 and 4

*(c)* 1 and 3

*(d)* 2 and 4

23. If $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size $n$ from $N(\mu, \sigma^2)$, where $\mu$ is known and we define

$$T = k \sum_{i=1}^{n} |X_i - \mu|$$

then for what value of $k$ will $T$ be an unbiased estimator of $\sigma$?

(a) $1 \cdot 25n$

(b) $0 \cdot 8n$

(c) $\frac{1 \cdot 25}{n}$

(d) $\frac{0 \cdot 8}{n}$

24. The sample values are drawn from a population with p.d.f. $f(x) = (1+\theta)x^{\theta}$; $0 < x < 1$, $\theta > 0$, and are 0·3, 0·4, 0·5, 0·6, 0·7, 0·8 and 0·9. The estimate of $\theta$ by the method of moments is

(a) 0·4

(b) 0·5

(c) 0·6

(d) 0·7

25. Consider a population with three kinds of individuals labelled 1, 2 and 3 occurring in the Hardy-Weinberg proportions $p(1, \theta) = \theta^2$, $p(2, \theta) = 2\theta(1-\theta)$ and $p(3, \theta) = (1-\theta)^2$, where $0 < \theta < 1$. If we observe a sample of three individuals and obtain $x_1 = 1$, $x_2 = 2$ and $x_3 = 1$, then MLE of $\theta$ is given by

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{5}{6}$

26. Consider the following statements :

1. The signed-rank test is a non-parametric analogue for the paired $t$-test.
2. The Behrens-Fisher problem has no exact solution.
3. The Mann-Whitney test is a non-parametric two-sample test.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

27. Consider the following statements :

1. Robustness of a test means relative insensitivity to violation of assumptions.
2. $t$-test should not be used for sample size 10 if the distribution is clearly skewed.
3. For sample size 100, $t$-test can be used even if the distribution is non-normal.

Which of the above statements is/are correct?

(a) 2 and 3 only

(b) 1 only

(c) 1 and 2 only

(d) 1, 2 and 3

**28.** If $T$ is an unbiased estimator for $\theta$, then $T^2$ is

*(a)* unbiased estimator for $\theta^2$

*(b)* biased estimator for $\theta^2$

*(c)* unbiased estimator for $(\theta^2 + 1)$

*(d)* biased estimator for $(\theta^2 + 1)$

**29.** If $X_1, X_2, X_3, \ldots, X_n$ is a random sample from a normal population $N(\mu, 1)$, the statistic

$$t = \frac{1}{n} \sum_{i=1}^{n} X_i^2$$

is an unbiased estimator of

*(a)* $\mu^2 + 1$

*(b)* $\mu + 1$

*(c)* $\mu$

*(d)* $2\mu + 1$

**30.** If $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a population with density

$$f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty$$

then the MLE of $\theta$ is given by

*(a)* $\overline{X}$

*(b)* median $(X_1, X_2, X_3, \ldots, X_n)$

*(c)* mode $(X_1, X_2, X_3, \ldots, X_n)$

*(d)* standard deviation

**31.** Let $X_1, X_2, X_3, \ldots, X_n$ be i.i.d. random variables with $E(X_i) = \theta$. Then for what value of $k$ will

$$T = \frac{k \sum_{i=1}^{n} iX_i}{n(n+1)}$$

be an unbiased estimator of $\theta$?

*(a)* 1

*(b)* 2

*(c)* 3

*(d)* 4

**32.** Let $X_1, X_2, X_3, \ldots, X_n$ be random variables from $N(\theta, \theta)$ for $\theta > 0$. Then

1. $T_n = \overline{X}_n S_n^2$ is an unbiased estimate of $\theta^2$, where $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$ and

$$S_n^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$$

2. $T_n$ is a consistent estimator of $\theta^2$

Which of the above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**33.** A normal distribution with unspecified mean and variance is fitted to the following frequency distribution :

| *Class interval* | *Frequency* |
|---|---|
| 10·5–20·5 | 3 |
| 20·5–30·5 | 5 |
| 30·5–40·5 | 12 |
| 40·5–50·5 | 25 |
| 50·5–60·5 | 23 |
| 60·5–70·5 | 15 |
| 70·5–80·5 | 7 |
| 80·5–90·5 | 2 |

What are the degrees of freedom for the corresponding chi-square test of goodness of fit?

*(a)* 3

*(b)* 4

*(c)* 5

*(d)* 6

**34.** The estimate of $\lambda$ for the exponential distribution $f(x; \lambda) = \lambda e^{-\lambda x}$ for $0 \le x < \infty$ by the method of moments is

*(a)* $\bar{x}$

*(b)* $\dfrac{1}{\bar{x}}$

*(c)* $\bar{x}^2$

*(d)* $\dfrac{1}{\sqrt{\bar{x}}}$

**35.** Let $P$ be the probability that a coin will fall head in a single toss in order to test the hypothesis $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and $H_0$ is rejected if more than three heads are obtained. The probability of type–I error is

*(a)* $\dfrac{3}{16}$

*(b)* $\dfrac{47}{128}$

*(c)* $\dfrac{81}{128}$

*(d)* $\dfrac{13}{16}$

**36.** If $x \ge 1$ is the critical region for testing $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$ on the basis of single observation from the population $f(x, \theta) = \theta e^{-\theta x}$, $0 \le x < \infty$, then the values of type–I and type–II errors will be respectively

*(a)* $\dfrac{e-1}{e}, \dfrac{1}{e^2}$

*(b)* $\dfrac{1}{e^2}, \dfrac{e-1}{e}$

*(c)* $\dfrac{e^2-1}{e^2}, \dfrac{1}{e}$

*(d)* $\dfrac{1}{e}, \dfrac{e^2-1}{e^2}$

**37.** Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from a normal population $N(\theta, 1)$. It is desired to test the null hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $(\theta_1 > \theta_0)$. Let a test reject $H_0$ when $\bar{x} > \frac{\theta_0 + \theta_1}{2}$. If $\alpha$ and $\beta$ are the type-I and type-II error probabilities of this test, then

*(a)* $\alpha < \beta$

*(b)* $\alpha = \beta$

*(c)* $\alpha > \beta$

*(d)* $\alpha + \beta > 1$

**38.** Consider the following statements :

Statement I :

UMVU estimator is always unique, if it exists.

Statement II :

UMVU estimator is provided by CR lower bound only.

Which one of the following is correct in respect of the above statements?

*(a)* Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

*(b)* Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I

*(c)* Statement I is true but Statement II is false

*(d)* Statement I is false but Statement II is true

**39.** For testing the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$, $\alpha$ and $\beta$ are the probabilities of type-I and type-II errors respectively. Under the SPRT criteria, the values of the constants $A$ and $B$ $(B < A)$ are respectively

*(a)* $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$

*(b)* $\frac{1-\beta}{\alpha}, \frac{\beta}{1-\alpha}$

*(c)* $\frac{\beta}{1-\alpha}, \frac{1-\beta}{\alpha}$

*(d)* $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$

**40.** A test in which decision about $H_0$ is taken after each successive observation is known as

*(a)* Bayes' test

*(b)* likelihood ratio test

*(c)* sequential probability ratio test

*(d)* Student's ratio test

**41.** Let there be a population having p.d.f.

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}; & \theta \le x < \infty \\ 0 \ ; & \text{otherwise} \end{cases}$$

The maximum likelihood estimator of $\theta$ based on a random sample of size $n$ is

*(a)* largest sample observation

*(b)* smallest sample observation

*(c)* sample median

*(d)* sample mode

**42.** If a sufficient statistic $t$ for $\theta$ exists, then

1. MLE will be a function of sufficient statistic if it is unique
2. it will have the invariance property
3. $\phi(t)$ will be MLE for $\phi(\theta)$, provided $\phi$ is a continuous function of $\theta$

Which of the above are correct?

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

**43.** Regarding optimum properties of MLE under some regulatory conditions, we can say that

1. the likelihood equation has a solution which converges in probability to true value $\theta_0$
2. a consistent solution of likelihood equation does not correspond to the maximum of the likelihood
3. the likelihood equation has one and only one consistent solution

Which of the above are correct?

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

**44.** Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample of size $n$ from a distribution with p.d.f.

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}; \quad 0 < x < \infty, \ \theta > 0$$

The maximum likelihood estimate of the median of the distribution is given by

*(a)* $\dfrac{\bar{x}}{\log_e 2}$

*(b)* $\dfrac{\log_e 2}{\bar{x}}$

*(c)* $\bar{x} \log_e 2$

*(d)* $\log_e 2$

**45.** The area of critical region depends on the

*(a)* size of type–I error

*(b)* size of type–II error

*(c)* value of statistic

*(d)* number of observations

**46.** Let $X_1, X_2$ be independently and identically distributed Poisson variables with parameter $\theta$. Then $X_1 + 2X_2$ is

*(a)* sufficient statistic for $\theta$

*(b)* not sufficient statistic for $\theta$

*(c)* minimal sufficient statistic for $\theta$

*(d)* complete sufficient statistic for $\theta$

**47.** Let $X_1, X_2, X_3, \ldots, X_n$ be independently and identically distributed random variables from a distribution with

$$f(x, \theta) = \begin{cases} (\theta+1)x^{\theta}\ ; & 0 < x < 1,\ \theta > 0 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

Which of the following statements are correct for parameter $\theta$ of the distribution?

1. The method of moments estimate of $\theta$ is given by $\frac{2\bar{x}-1}{1-\bar{x}}$, where $\bar{x}$ is the sample mean.

2. The estimate of $\frac{1}{\theta}$ obtained by the method of moments is $\frac{1-H}{H}$, where $H$ is the sample harmonic mean.

3. The maximum likelihood estimate of $\theta$ is $-\left(\frac{1+\log G}{\log G}\right)$, where $G$ is the sample geometric mean.

Select the correct answer using the code given below.

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

**48.** Regularity conditions of Cramer-Rao inequality are related to

*(a)* integrability of functions

*(b)* differentiability of functions

*(c)* both integrability and differentiability of functions

*(d)* neither integrability nor differentiability of functions

**49.** The decision criteria in SPRT depend on the functions of

*(a)* type–I error

*(b)* type–II error

*(c)* both type–I and type–II errors

*(d)* neither type–I nor type–II error

**50.** For testing $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = 2\theta_0$, based on a single random observation $y$ from the population with p.d.f.

$$f(x, \theta) = \theta e^{-\theta x}; \quad \theta > 0, \ 0 \le x < \infty$$

the critical region for the hypothesis is $y \ge \frac{2}{\theta_0}$. Then type–I error and type–II error respectively will be

*(a)* $e^{-6}$ and $e^{-4}$

*(b)* $e^{-4}$ and $e^{-2}$

*(c)* $e^{-2}$ and $e^{-4}$

*(d)* $e^{-4}$ and $e^{-6}$

**51.** Let $x_1, x_2, x_3, \ldots, x_n$ be a random sample from the population with p.d.f.

$$f(x, \theta) = \theta x^{\theta - 1}; \quad 0 < x < 1, \ \theta > 0$$

Which one of the following is correct?

*(a)* $t_1 = \prod_{i=1}^{n} x_i$ is sufficient estimator for $\theta$

*(b)* $t_1 = \prod_{i=1}^{n} x_i$ is not sufficient estimator for $\theta$

*(c)* $t_1 = \sum_{i=1}^{n} x_i$ is sufficient estimator for $\theta$

*(d)* There does not exist any sufficient estimator for $\theta$

**52.** Let $X_1, X_2, X_3, \ldots, X_n$ be i.i.d. $U(0, \theta)$ variates. Let

$$Y = \max(X_1, X_2, X_3, \ldots, X_n)$$

Then the unbiased estimator of $(3\theta^3 + 5\theta^5)$ is given by

*(a)* $(n+3)Y^3 + (n+5)Y^5$

*(b)* $\left(\frac{n+3}{n}\right)Y^3 + \left(\frac{n+5}{n}\right)Y^5$

*(c)* $3(n+3)Y^3 + 5(n+5)Y^5$

*(d)* $3\left(\frac{n+3}{n}\right)Y^3 + 5\left(\frac{n+5}{n}\right)Y^5$

**53.** Let $X$ be a random variable following binomial distribution with parameters $n = 10$ and $p$. Suppose $x$ is a sample value of size 1. The test is that if $x \leq 3$, then reject $H_0 : p = \frac{1}{2}$ and accept $H_1 : p = \frac{1}{4}$. The size of the critical region is

(a) $\frac{1}{8}$

(b) $\frac{9}{64}$

(c) $\frac{5}{32}$

(d) $\frac{11}{64}$

**54.** Let $X$ have a p.d.f. of the form $f(x, \theta) = \theta x^{\theta - 1}$, $0 < x < 1$. To test $H_0 : \theta = 1$ against the alternative $H_1 : \theta = 2$, we use a random sample $X_1$, $X_2$ of size $n = 2$ and define the critical region

$$\omega = \left\{ (x_1, x_2) : \frac{3}{4x_1} \leq x_2 \right\}$$

The size of the type–I error is

[Given, $\ln(0 \cdot 75) = -0 \cdot 3$]

(a) $0 \cdot 01$

(b) $0 \cdot 025$

(c) $0 \cdot 05$

(d) $0 \cdot 1$

**55.** A random variable takes the values 1, 2 and 3 with probabilities $\theta^2$, $2\theta(1-\theta)$ and $(1-\theta)^2$ respectively with $0 < \theta < 1$. However the observed frequencies of 1, 2 and 3 are $n_1$, $n_2$ and $n_3$ respectively. The maximum likelihood estimator of $\theta$ is given by

(a) $\dfrac{2n_1 - n_2}{2(n_1 + n_2 + n_3)}$

(b) $\dfrac{2n_1 + n_2}{2(n_1 + n_2 + n_3)}$

(c) $\dfrac{n_1 - 2n_2}{2(n_1 + n_2 + n_3)}$

(d) $\dfrac{n_1 + 2n_2}{2(n_1 + n_2 + n_3)}$

**56.** Let $X_1, X_2, X_3, \ldots, X_{2n}$ be i.i.d. random variables from $N(\mu, \sigma^2)$. Then the value of $k$ for which

$$t_{2n}(X) = k[(X_1 - X_2)^2 + (X_3 - X_4)^2 + \ldots + (X_{2n-1} - X_{2n})^2]$$

is an unbiased estimator of $\sigma^2$ is given by

(a) $\frac{1}{n}$

(b) $\frac{1}{2n}$

(c) $\frac{1}{3n}$

(d) $\frac{1}{4n}$

**57.** Let $T$ be MVU estimator of $g(\theta)$. Let $T_1$ and $T_2$ be unbiased estimators of $g(\theta)$ with efficiencies $e_1 = 0 \cdot 4$ and $e_2 = 0 \cdot 9$ respectively. Let $\rho$ be the correlation coefficient between $T_1$ and $T_2$ whose values lie in the interval

*(a)* $0 \cdot 335 \leq \rho \leq 0 \cdot 825$

*(b)* $0 \cdot 345 \leq \rho \leq 0 \cdot 835$

*(c)* $0 \cdot 355 \leq \rho \leq 0 \cdot 845$

*(d)* $0 \cdot 365 \leq \rho \leq 0 \cdot 855$

**58.** If $T_1$ and $T_2$ be unbiased estimators of $g(\theta)$ with variances $\sigma_1^2 = 1$ and $\sigma_2^2 = 4$ and Corr $(T_1, T_2) = \frac{1}{4}$, then the values of $W_1$ and $W_2$ for which $T = W_1 T_1 + W_2 T_2$ is BLUE for $g(\theta)$ are respectively

*(a)* $\frac{7}{8}$ and $\frac{1}{8}$

*(b)* $\frac{5}{8}$ and $\frac{3}{8}$

*(c)* $\frac{1}{2}$ and $\frac{1}{2}$

*(d)* $\frac{1}{8}$ and $\frac{7}{8}$

**59.** Which of the following statements is/are correct?

1. Cramer-Rao inequality gives an upper bound for the variance of an unbiased statistic.

2. Rao-Blackwellization means conditioning on a sufficient statistic to reduce variance.

Select the correct answer using the code given below.

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**60.** Consider the following statements :

1. If $T_n$ is consistent for $\theta$, then for any function $f$, $f(T_n)$ is consistent for $f(\theta)$.

2. Method of moments gives consistent estimators.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**61.** Consider the testing problem for $N(\theta, 1)$ :

$$P_1 \ : \ H_0 : \theta = \theta_0 \text{ vs. } H_1 : \theta > \theta_0$$

$$P_2 \ : \ H_0 : \theta = \theta_0 \text{ vs. } H_1 : \theta \neq \theta_0$$

The UMP level $\alpha$ test exists

*(a)* for $P_1$ but not for $P_2$

*(b)* for $P_2$ but not for $P_1$

*(c)* for both $P_1$ and $P_2$

*(d)* neither for $P_1$ nor for $P_2$

**62.** Let $\frac{X}{\sigma_1^2}$ and $\frac{Y}{\sigma_2^2}$ be two independent chi-square variates with 2 and 6 degrees of freedom respectively. An unbiased estimator of $\frac{\sigma_1^2}{\sigma_2^2}$ is given by

*(a)* $\frac{X}{Y}$

*(b)* $\frac{2X}{Y}$

*(c)* $\frac{3X}{Y}$

*(d)* $\frac{4X}{Y}$

**63.** Let $X$ be a binomial random variable with parameters $n$ and $\theta$. Let

$$t_n(X) = \frac{X - \frac{\sqrt{n}}{2}}{n - \sqrt{n}}$$

Then consider the following statements :

1. $t_n(X)$ is biased estimator of $\theta$.
2. $t_n(X)$ is consistent estimator of $\theta$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**64.** A sample is drawn from the population

$$f(x;\ \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}, \quad x \geq 0$$

and the values of first and second order sample moments about origin are 10 and 150 respectively. Then the estimators of $\alpha$ and $\beta$ obtained by method of moments are

*(a)* $\hat{\alpha} = 2$ and $\hat{\beta} = \frac{1}{5}$

*(b)* $\hat{\alpha} = \frac{1}{2}$ and $\hat{\beta} = 5$

*(c)* $\hat{\alpha} = 2$ and $\hat{\beta} = 5$

*(d)* $\hat{\alpha} = \frac{1}{2}$ and $\hat{\beta} = \frac{1}{5}$

**65.** Consider the following statements :

Statement I :

SPRT sometimes provides a decision.

Statement II :

Probability of terminating SPRT is one.

Which one of the following is correct in respect of the above statements?

*(a)* Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

*(b)* Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I

*(c)* Statement I is true but Statement II is false

*(d)* Statement I is false but Statement II is true

**66.** Consider the following statements :

1. Indian Statistical System is decentralized by federal structure as also by subjects.
2. State Statistical System is centralized.
3. Statistical coordination is done by CSO.

Which of the above statements are correct?

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

**67.** Consider the following statements :

1. National Statistical Commission has the responsibility of statistical audit.
2. NSSO collects data by sample surveys.
3. Censuses are conducted by RGI only.

Which of the above statements are correct?

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

**68.** Indian Statistical System is supported by which legislative framework?

1. The Collection of Statistics Act, 2008
2. The Registration of Births and Deaths Act, 1969
3. The Census Act, 1948

Select the correct answer using the code given below.

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

69. Consider the following :

1. National Family Health Survey— Ministry of Health and Family Welfare
2. Civil Registration System— Registrar General and Census Commissioner of India
3. Consumer Expenditure Survey— National Sample Survey Office

Which of the above are correctly matched?

*(a)* 1 and 2 only

*(b)* 2 and 3 only

*(c)* 1 and 3 only

*(d)* 1, 2 and 3

70. Which of the following is/are the publication/publications of National Accounts Division of CSO in the MOSPI ?

1. National Accounts Statistics
2. Input-Output Transactions Table
3. Economic Census

Select the correct answer using the code given below.

*(a)* 1 only

*(b)* 1 and 2

*(c)* 1 and 3

*(d)* 2 and 3

71. Agriculture Census data is collected in Land Records States through

1. complete enumeration of units
2. Khasra registers retabulation
3. sampling techniques

Which of the above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* 1 and 2

*(d)* 1 and 3

72. Which one of the following is correct in respect of Gross Enrolment Ratio (GER) in Grade–I ?

*(a)* $\text{GER} = \dfrac{\text{Total Grade–I enrolment}}{\text{Population of age 6 years}} \times 100$

*(b)* $\text{GER} = \dfrac{\text{New entrants in Grade–I}}{\text{Population of age 6 years}} \times 100$

*(c)* $\text{GER} = \dfrac{\text{New entrants of age 6 years in Grade–I}}{\text{Population of age 6 years}} \times 100$

*(d)* $\text{GER} = \dfrac{\text{Total Grade–I enrolment}}{\text{Population of age 6 years}} \times 1000$

73. The reference period in employment/unemployment surveys by NSSO is

1. one year
2. two weeks
3. each day of reference week
4. one week

Which of the above is/are correct?

*(a)* 1, 2 and 3

*(b)* 1, 3 and 4

*(c)* 1 only

*(d)* 3 only

74. Which one of the following equations is valid?

*(a)* Promotion rate + Repetition rate + Dropout rate = 100

*(b)* Promotion rate + Repetition rate – Dropout rate = 100

*(c)* Promotion rate – Repetition rate + Dropout rate = 100

*(d)* None of the above

75. The Agriculture Census in India is conducted

*(a)* annual basis

*(b)* quinquennial basis

*(c)* decadal basis

*(d)* ad hoc basis

76. Consider the following :

1. The Prevention of Food Adulteration Act, 1954
2. The Medical Termination of Pregnancy Act, 1971
3. The Food Safety and Standards Act, 2006

Which of the above comes/come under the jurisdiction of MOHFW ?

*(a)* 2 only

*(b)* 1 and 3 only

*(c)* 1 and 2 only

*(d)* 1, 2 and 3

**77.** Sample registration for collecting vital statistics is

*(a)* a fixed panel survey

*(b)* a cross-sectional survey

*(c)* both fixed panel survey and cross-sectional survey

*(d)* neither fixed panel survey nor cross-sectional survey

**78.** If the price index for the year 2015 is 110·3 and the price index for the year 2005 is 98·4, then what is the purchasing power of money (in ₹) of 2005 compared to 2015?

*(a)* 1·12

*(b)* 1·25

*(c)* 1·35

*(d)* 0·89

**79.** Consumer price index numbers reveal the state of

*(a)* inflation

*(b)* deflation

*(c)* both inflation and deflation

*(d)* neither inflation nor deflation

**80.** The following table gives the frequency distribution of number of live births born to women in the age group 15–45 years :

| *Age group* | *Number of women* | *Number of live births* |
|---|---|---|
| 15–19 | 20000 | 600 |
| 20–24 | 18000 | 1200 |
| 25–29 | 14000 | 800 |
| 30–45 | 8000 | 96 |

The value of General Fertility Rate (GFR) based on the above data is

*(a)* 44·933

*(b)* 89·866

*(c)* 449·33

*(d)* 898·66

★★★

SPACE FOR ROUGH WORK

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