Combined Geo-Scientist (Main) Examination, 2024

SGSE-P-GPH

GEO-PHYSICS

Paper – I

*Time Allowed : **Three Hours***

*Maximum Marks : **200***

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **TEN** questions divided in **TWO** sections.*

*Candidate has to attempt **SIX** questions in all.*

*Question Nos. **1** and **6** are **compulsory**.*

*Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.*

The number of marks carried by a question / part is indicated against it.

Neat sketches are to be drawn to illustrate answers, wherever required. They shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer (QCA) Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) Derive Adams–Williamson equation and its application in Geophysics. Also give variation of Body-wave velocity (α) in the Earth's interior according to Earth Model IASP91. *10*

(b) State Fermat's principle and derive Snell's law using the same principle for geometry of incident and refracted ray on horizontal interface. *10*

(c) Write down the condition under which the magnetic dipole is presumed to be a point or isolated pole. Draw the magnetic effects in terms of H, Z and F profiles due to an isolated pole for I = 45°, where I is the field inclination. *10*

(d) The diagonal elements of a covariance matrix computed for a linearized inverse problem having model parameters m_1, m_2, m_3, m_4, m_5 are 49, 15, 3, 200, 40 respectively. Compute the standard deviation in the estimation of model parameter m_4. *10*

Q2. (a) (i) Give Kepler's laws of planetary motion relating the empirical laws as expressions of fundamental physical laws. *6*

(ii) Define the following terms as parameters of an elliptical orbit :

(1) Aphelion and Perihelion *2*

(2) Perigee and Apogee *2*

(b) Explain, briefly, Newton's law of gravitation. If a satellite is orbiting the Earth 250 km above the surface, what acceleration due to gravity does it experience ? Given that the radius of the Earth is $6{\cdot}38 \times 10^6$ m and the mass of the Earth is $5{\cdot}98 \times 10^{24}$ kg. *10*

(c) (i) Find the general formula for inverse of 2×2 square matrix. *4*

(ii) Compute the inverse of the 3×3 square matrix given as *6*

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 1 & -2 & -2 \\ -3 & 3 & 2 \end{bmatrix}.$$

Q3. (a) What forces are related to Isostasy ? A large area of continent consists of 30 km of crust with an average density of 2·8 g/cc and over 90 km thick material with density of 3·1 g/cc. It is covered with a 1·6 km thick layer of ice (density 0·9 g/cc) and is in isostatic equilibrium. Then the ice melts. After equilibrium has been regained, by how much has the rock surface of the continent changed ? (Density of asthenosphere is 3·2 g/cc). *10*

(b) Describe the distortion of current flow at the plane interface using suitable sketch. Explain image theory approach in the case of electrical prospecting. Compute the value of reflection coefficient, k, for ρ_1 = 10 ohm-m and ρ_2 = 40 ohm-m, where ρ_1 and ρ_2 are the resistivities of the two media. *10*

(c) Discuss the following in seismic wave propagation :

(i) Seismic Wave Attenuation *5*

(ii) Seismic Wave Dispersion *5*

Q4. (a) What causes Geomagnetism ? Give the definition of geomagnetic elements and the relationship of Cartesian coordinates (X_e, Y_e, Z_e) and spherical polar (F_e, D, I) sets of Earth's magnetic elements. *10*

(b) (i) Explain the role of eigenvalues and eigenvectors to solve the inverse problem. *3*

(ii) Find out the eigenvalues and eigenvectors of a 2 × 2 square matrix *7*

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

(c) Define ray parameter for spherical earth. Using ray parameter, derive the expression for travel time and epicentral distance. *10*

Q5. (a) (i) Draw focal mechanism for the following :

(1) Normal-faulting earthquake *2*

(2) Thrust-faulting earthquake *2*

(ii) Describe the following seismic phases :

(1) PKJKP *2*

(2) PKiKP *2*

(iii) Describe the Gutenberg–Richter relationship in seismology and its applications. 2

(b) Consider a thin wire carrying a current. Show the relation between electric and magnetic fields using suitable sketch. Explain Maxwell's equations for the electromagnetic fields. 10

(c) Consider a linear system $A_x = b$,

where $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, and $b = \begin{bmatrix} -1 \\ 7 \\ 7 \end{bmatrix}$

(i) Can we use Steepest Descent (SD) method for solving the above linear system ? 4

(ii) If yes, compute three iterations by SD method starting with $x_0 = [0, 0, 0]^T$. 6

SECTION B

Q6. (a) Find the solution of the following differential equation

$$\frac{d^2y}{dt^2} - y = 0, \; y = y(t),$$

when y satisfies the initial conditions $y(0) = 2$ and $\frac{dy}{dt}(0) = \alpha$, where α is a positive constant. Then find the value of α so that the solution approaches zero as $t \to \infty$. *10*

(b) Suppose we have some charge and current configuration in space which, at time t, produces an electric field $\vec{E}(\vec{r}, t)$ and a magnetic field $\vec{B}(\vec{r}, t)$. In the next instant $(t + dt)$, the charges move around a bit. Show that the work done per unit time, $\frac{dW}{dt}$, by the electromagnetic forces acting on the charges is equal to the rate of decrease in the energy stored in the fields contained in the volume V enclosed by the surface S, less the energy that flowed out through the surface S. *10*

(c) Starting from Maxwell equations prove :

(i) Coulomb's Law *5*

(ii) Continuity Equation *5*

(d) Find the exact altitude of a GPS satellite that has an orbital period equal to precisely one-half of a sidereal day. Use a value of mean earth radius $r_c = 6378{\cdot}14$ km and sidereal day length of 23 hours 56 minutes 4·1 seconds. *10*

Q7. (a) Calculate and determine the Laurent series for the function

$$f(z) = \frac{1}{z(z+5)},$$

valid for the region $\{z : |z| < 5\}$. *10*

(b) Write down the expressions for the real electric ($\vec{E}$) and magnetic ($\vec{B}$) fields for a monochromatic plane wave of amplitude E_0, frequency ω and the phase angle zero that is travelling in the direction from the origin to the point (1, 1, 1) with polarization parallel to the xz-plane. *10*

(c) Show that the electromagnetic potentials in uniform electric and magnetic fields may be expressed as

$$\phi = \vec{E} \cdot \vec{r} \text{ and } \vec{A} = \frac{1}{2}(\vec{B} \times \vec{r}),$$

where $\vec{r}$ is the position vector of the point under consideration. *10*

Q8. (a) Using the substitution $z = e^{i\theta}$, where θ is real and positive, and the residue theorem, evaluate the integral *10*

$$I = \int_0^{2\pi} \frac{1}{(2 + \sin\theta)^2} d\theta.$$

(b) A spherical shell of radius R, carrying a uniform surface charge density σ, is set spinning at angular velocity ω. Calculate its magnetic dipole moment. Show that, for r >> R, the magnetic vector potential of the shell is that of a perfect dipole. *10*

(c) The radiation resistance of an antenna is 80 Ω and loss resistance is 10 Ω. Find its directivity if power gain is 20. *10*

Q9. (a) Using the laws of transformation of the electric field $\vec{E}$ and the magnetic field $\vec{B}$, under Lorentz transformations, show that the combination $(\vec{E}^2 - c^2 \vec{B}^2)$ is relativistically invariant. *10*

(b) In a Young's double-slit experiment, the separation between the second-order bright fringe and the central bright fringe, Y, on a flat screen is 0·0180 m, when the light has a wavelength of 425 nm. Assume that the angles, θ, that locate the fringes on the screen are so small that $\sin\theta \approx \tan\theta$. Calculate the separation Y when the wavelength of light is 585 nm. *10*

(c) What is GPS augmentation ? Why is it required ? Name a few of the GPS augmentation systems. *10*

Q10. (a) What would be the height of the atmosphere if the air density *10*

(i) were uniform,

(ii) decreased linearly to zero with height ?

Assume that at sea level the air pressure is 2·0 atm and the air density is 1·3 kg/m^3. [Given g = 9·8 m/s^2]

(b) Find the Laplace transform of the function *10*

$$F(t) = \frac{e^{at} - 1}{a}.$$

(c) For some spherically symmetric distribution of charges, the potential on the surface of radius R is given to be V_0. Calculate the potential in the region r > R. Also calculate the potential for r < R, when no charges are present inside the sphere. *10*