GEO-PHYSICS

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **TEN** questions divided in **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Question Nos. **1** and **6** are compulsory.

Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION—A

1\. *(a)* What is the relationship between the angle of inclination of the remanent magnetisation and the magnetic latitude at the time of the magnetisation of the rock? Assuming that the angle of inclination is 60°, calculate the magnetic latitude. 10

(b) The P-wave from an earthquake arrives at a seismic station at 13 hours 19 minutes 58·9 seconds and the S-wave arrives at 13 hours 20 minutes 4·7 seconds. Assuming that the P-wave velocity is 5·6 km s^{-1} and that Poisson's ratio is 0·25, calculate the time at which the earthquake occurred. 10

(c) A cylindrical conductor of radius r_0 has internal magnetic field intensity

$$\vec{H} = \frac{10^3}{r}\left[\frac{1}{2b^2}\sin br - \frac{r}{4b}\cos br\right]\vec{b}_\phi$$

where $b = \frac{\pi}{2r_0}$. Calculate the total current flowing in the conductor if radius $r_0 = 1$ cm. 10

(d) A gravity profile of 25 points indicates the presence of an ore body of horizontal cylindrical shape. Formulate a non-linear inverse problem to determine the depth (z), dimension (R) and density contrast (ρ_c) for the ore body. Write the steps, along with necessary equations, to determine z, R and ρ_c using Gauss-Newton method. 3+7=10

2\. *(a)* A mountain range of 3·5 km height is in isostatic equilibrium with a continental crust of thickness 38 km. During a period of crustal shortening, 2·5 km of material is added to the mountain range, leading to an increase in the height of the mountain. Calculate the new height of the mountain, after the isostatic equilibrium is attained. Assume the density of the continental crust to be 2900 kg/m^3 and the mantle to be 3400 kg/m^3. 10

(b) The igneous rock of susceptibility 0·18 (SI) intrudes the host sedimentary rock of susceptibility 3×10^{-4} (SI) and forms an anticlinal horizontal cylinder.

(i) Assuming that the rocks are magnetised in the vertical direction by the vertical component of the magnetic field of strength 38000 nT, calculate the induced magnetisation contrast between the igneous and sedimentary rocks. 5

(ii) Assuming that the radius of the horizontal cylinder is one-third the depth to the cylindrical axis, calculate the maximum value of the vertical field anomaly over the cylinder. 5

(Given, $\mu_0 = 4\pi \times 10^{-7}$ H/m)

(c) Two waves with frequencies of 10 Hz and 100 Hz propagate through a medium of velocity 2 km s^{-1} and absorption coefficient, $\alpha = 0{\cdot}5$ dB λ^{-1} (λ is wavelength and dB is decibel). What is the wave attenuation in decibels due to absorption for both waves? What is the inference you draw from your results thus obtained? 8+2=10

3. *(a)* Obtain the inverse of the following matrix using Singular Value Decomposition (SVD) of the matrix : 10

$$\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$$

(b) Define Apparent Polar Wander (APW) path in the context of continental drift theory. How do APW paths of the continents provide evidence to the continental drift? What are the limitations of the APW studies of Precambrian rocks? 10

(c) Calculate the bulk modulus (K), shear modulus (μ) and Poisson's ratio (σ) for the lower crust using the values for P-wave velocity (V_P) as 7·4 km s^{-1}, S-wave velocity (V_S) as 4·3 km s^{-1} and density (ρ) as 3100 kg m^{-3}. 10

4. *(a)* The $d_{1/4}$ is the half-width value at $\frac{1}{4}$th of the maximum gravity value measured over a buried sphere of uniform density ρ. If z is the depth from the surface to the centre of the sphere, derive the expression for $d_{1/4}$ in terms of z. 10

(b) Write an expression for minimum norm solution of a linear inverse problem $d = Gm$ (d : data matrix; m : model parameters matrix). Use the expression to solve the equation $m_1 + m_2 = 2$, where m_1 and m_2 are model parameters. What are model resolution and data resolution matrices for the solution you have obtained? 2+4+4=10

(c) The normal gravity formula is given by

$$g_n = g_e (1 + \beta_1 \sin^2 \lambda + \beta_2 \sin^2 2\lambda)$$

where $g_e = 9{\cdot}780327\ \text{m/s}^2$, $\beta_1 = 5{\cdot}30244 \times 10^{-3}$ and $\beta_2 = -5{\cdot}8 \times 10^{-6}$.

(i) Derive an expression for the change of gravity value with latitude. 5

(ii) What will be the change in the gravity values in milligals per kilometre along northward direction at latitude 30°? [Use radius of the earth = 6371 km] 5

5. *(a)* *(i)* With the help of a neat sketch, differentiate between a forward problem and an inverse problem in terms of model, model parameters and data. Formulate a linear inverse problem $d = Gm$ (d : data matrix; m : model parameters matrix) for the given 10 temperature values ($T_1, T_2, \cdots, T_{10}$) at 10 depth levels ($z_1, z_2, \cdots, z_{10}$) by assuming the model $T = m_1 e^{m_2 z}$ (m_1 and m_2 are model parameters). 2+5=7

(ii) How will you solve the linear inverse problem formulated above using a least square solution? 3

(b) With the help of a neat sketch, describe the elastic rebound model that explains the origin of a tectonic earthquake. 10

(c) Describe the Gauss's law for magnetic fields. What are the geophysical constraints provided by the Gauss's law for magnetic field due to localised magnetisation distribution buried in the subsurface? Assume the lateral extent of the magnetic survey to be large compared to the size of the magnetisation distribution (sources). 10

SECTION—B

6. (a) Given a function $a(x, y) = y^3 - 3x^2y$.

(i) Is this function harmonic? 5

(ii) If yes, find the conjugate function of $a(x, y)$. 5

(b) Using the Laplace equation, find the potential and electric field intensity in the gap between two parallel plates charged with ϕ_1 and ϕ_2 potentials. 10

(c) For free space, write down Maxwell's equations in differential form and show that E_x indeed obeys the three-dimensional wave equation with speed

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$ 10

(d) How does the maximum usable frequency (f_{muf}) of radio wave propagation vary with the height (h) of the layer and propagation distance (D) in shorter distance communication (up to 500 km)? 10

7. (a) Find the residue of $\dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 1$. 10

(b) Expand $G(z) = \dfrac{1}{z(z-2)}$ in terms of Laurent series by the use of geometric series.

The expansion is to be done in—

(i) $0 < |z| < 2$

(ii) $2 < |z| < \infty$ 5+5=10

(c) (i) One of the two wavelengths emitted by a light source is 5400 Å. When the source is used in a double-slit interference experiment, the separation between the '4th' order bright fringes becomes 1·5 cm. If the distance between the source and the screen is 1·5 m and the distance between the slits is 0·24 mm, find the wavelength of other emitted line. 8

(ii) Explain why the central Newton's ring of reflected light is dark. 2

8. (a) Find the Fourier series for the periodic function

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$ 10

(b) A particle of charge q moves in a circle of radius a at constant angular velocity ω. Assume that the circle lies in the xy-plane, centred at the origin, and at time $t = 0$, the charge is at $(a, 0)$ on the positive x-axis. Find the Lienard-Wiechert potentials for points on the z-axis. 10

(c) *(i)* Characterise the composition of the earth's atmosphere. 5

(ii) Explain the term 'pressure gradient force' of the atmosphere. 2

(iii) Sketch the temperature profile in the earth's atmosphere and name the boundary layers associated with temperature maxima and minima. 3

9. *(a)* By using recurrence relation, show that

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1}$$ 10

(b) *(i)* What is 'geostationary' satellite and why is it called geostationary? 3

(ii) Discuss the benefits and limitations of geostationary satellites. 7

(c) An electric field arising due to stationary charge is given by

$$E = 300i + 400j \text{ volt/metre}$$

How will you represent this field in a frame moving relative to charges with velocity $v = (4i + 3j) \times 10^6$ metre/s? 10

10. *(a)* Calculate the geopotential height of the 1000 hPa pressure surface when the pressure at sea level is 1014 hPa. The scale height of the atmosphere may be taken as 8 km. 10

(b) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, if—

(i) the current is uniformly distributed over the outside surface of the wire;

(ii) the current is distributed in such a way that J is proportional to s, the distance from the axis. 4+6=10

(c) Determine the reflection coefficient and the transmission coefficient of an electric field wave traveling in air and incident normally on a boundary between air and dielectric having permeability $= \mu_0$ and permittivity, $\varepsilon_r = 16$. 10

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