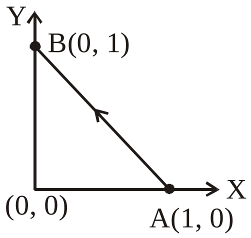
FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

1. Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is : (all quantities are in SI units)



(1) $\frac{3}{2}$ (2) 1 (3) 2 (4) $\frac{1}{2}$

NTA Ans. (2)

Sol. $W = \int_{\vec{r}_i}^{\vec{r}_g} \vec{F}.d\vec{r}$

$$W = \int_1^0 -xdx + \int_0^1 ydy$$

$$W = \left.\frac{-x^2}{2}\right|_1^0 + \left.\frac{y^2}{2}\right|_0^1$$

$$= -\left(\frac{0^2}{2} - \frac{1^2}{2}\right) + \left(\frac{1^2}{2} - \frac{0^2}{2}\right)$$

W = 1J

2. A quantity f is given by $f = \sqrt{\frac{hc^5}{G}}$ where c is speed of light, G universal gravitational constant and h is the Planck's constant. Dimension of f is that of :

(1) Momentum (2) Area

(3) Energy (4) Volume

NTA Ans. (3)

Sol. $[h] = M^1L^2T^{-1}$

$[C] = L^1T^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$$[f] = \sqrt{\frac{M^1L^2T^{-1} \times L^5T^{-5}}{M^{-1}L^3T^{-2}}} = M^1L^2T^{-2}$$

3. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body :

(1) starts moving in an elliptical orbit around the planet.

(2) continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Escapes from the Planet's Gravitational field.

NTA Ans. (1)

Sol. Initially, the body of mass m is moving in a circular orbit of radius R. So it must be moving with orbital speed.

$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision, let the combined mass moves with speed v_1

$$mv_0 + \frac{m}{2}\frac{v_0}{2} = \left(\frac{3m}{2}\right)v_1$$

$$v_1 = \frac{5v_0}{6}$$

Since after collision, the speed is not equal to orbital speed at that point. So motion cannot be circular. Since velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field.

So their motion will be elliptical around the planet.

4. The electric fields of two plane electromagnetic plane waves in vacuum are given by

$$\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx) \quad \text{and}$$

$$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$$

At t = 0, a particle of charge q is at origin with a velocity $\vec{v} = 0.8c\hat{j}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is :

(1) $E_0q(-0.8\hat{i} + \hat{j} + \hat{k})$

(2) $E_0q(0.8\hat{i} - \hat{j} + 0.4\hat{k})$

(3) $E_0q(0.8\hat{i} + \hat{j} + 0.2\hat{k})$

(4) $E_0q(0.4\hat{i} - 3\hat{j} + 0.8\hat{k})$

NTA Ans. (3)

Sol. $\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$

Its corresponding magnetic field will be

$$\vec{B}_1 = \frac{E_0}{c}\hat{k}\cos(\omega t - kx)$$

$$\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$$

$$\vec{B}_2 = \frac{E_0}{c}\hat{i}\cos(\omega t - ky)$$

Net force on charge particle

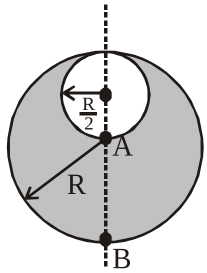
$$= q\vec{E}_1 + q\vec{E}_2 + q\vec{v}\times B_1 + q\vec{v}\times\vec{B}_2$$

$$= qE_0\hat{j} + qE_0\hat{k} + q(0.8c\hat{j})\times\left(\frac{E_0}{c}\hat{k}\right) + q(0.8c\hat{j})\times\left(\frac{E_0}{c}\hat{i}\right)$$

$$= qE_0\hat{j} + qE_0\hat{k} + 0.8qE_0\hat{i} - 0.8qE_0\hat{k}$$

$$\vec{F} = qE_0[0.8\hat{i} + 1\hat{j} + 0.2\hat{k}]$$

5. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is :



(1) $\frac{18}{54}$ (2) $\frac{21}{34}$ (3) $\frac{17}{54}$ (4) $\frac{18}{34}$

NTA Ans. (4)

Sol. Fill the empty space with +ρ and –ρ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

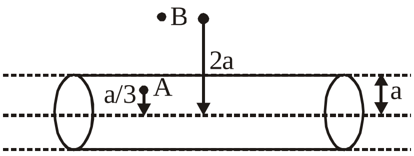
$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

6. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and 2a, respectively from the axis of the wire is :

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 2

NTA Ans. (1)

Sol.



Let current density be J.

$\therefore$ Applying Ampere's law.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \Rightarrow B_A 2\pi\frac{a}{3} = \mu_0 J\pi\left(\frac{a}{3}\right)^2$$

$$\therefore \quad B_A = \frac{\mu_0 Ja}{6}$$

Similarly, $B_B = \frac{\mu_0 Ja}{4}$

$$\therefore \quad \frac{B_A}{B_B} = \frac{\mu_0 Ja \times 4}{\mu_0 J6a} = \frac{2}{3}$$

7. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats $(C_V^A \text{ and } C_V^B)$ of gas A and B, respectively is :

(1) 7 : 9 (2) 5 : 7 (3) 3 : 5 (4) 5 : 9

NTA Ans. (2)

Sol. Degree of freedom of a diatomic molecule if vibration is absent = 5

Degree of freedom of a diatomic molecule if vibration is present = 7

$$\therefore \quad C_v^A = \frac{f_A}{2}R = \frac{5}{2}R \ \& \ C_v^B = \frac{f_B}{2}R = \frac{7}{2}R$$

$$\therefore \quad \frac{C_v^A}{C_v^B} = \frac{5}{7}$$

8. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is added to its energy, the wavelength become λ/2. Value of ΔE, is :

(1) 2E (2) E (3) 3E (4) 4E

NTA Ans. (3)

Sol. Given, de-Broglie wavelength = $\frac{h}{\sqrt{2mE}} = \lambda$

Also, $\frac{h}{\sqrt{2m(E+\Delta E)}} = \frac{\lambda}{2}$

$\therefore \quad \frac{E+\Delta E}{E} = 4 \Rightarrow \Delta E = 3E.$

9. If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is :

(1) 0.001 mm (2) 0.001 cm

(3) 0.02 mm (4) 0.01 cm

NTA Ans. (2)

Sol. Given on six rotation, reading of main scale changes by 3mm.

$\therefore$ 1 rotation corresponds to $\frac{1}{2}$mm

Also no. of division on circular scale = 50.

$\therefore$ Least count of the screw gauge will be

$\frac{0.5}{50}$mm = 0.001 cm.

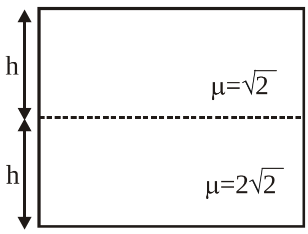
10. A vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be :

(1) $\frac{h}{\sqrt{2}}$ (2) $\frac{3}{4}h\sqrt{2}$

(3) $\frac{h}{2\left(\sqrt{2}+1\right)}$ (4) $\frac{h}{3\sqrt{2}}$

NTA Ans. (2)

Sol.



For near normal incidence,

$$h_{app} = \frac{h_{actual}}{\left(\frac{\mu_{in}}{\mu_{ref.}}\right)}$$

$$\therefore \quad h_{apparent} = \frac{\frac{h}{\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)}+h}{\frac{\sqrt{2}}{1}} = \frac{3h}{2\sqrt{2}} = \frac{3}{4}h\sqrt{2}$$

11. Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to :

(1) 1.8eV (2) 1.1eV

(3) 0.8eV (4) 1.6eV

NTA Ans. (3)

Sol. Let the work function be ϕ.

$$\therefore \quad KE_{max} = \frac{hc}{\lambda} - \phi$$

Again, $R_{max} = \frac{\sqrt{2mKE_{max}}}{qB} = \frac{\sqrt{2m\left(\frac{hc}{\lambda}-\phi\right)}}{qB}$

$$\therefore \quad \frac{R_{max}^2 q^2 B^2}{2m} = \frac{hc}{\lambda} - \phi$$

$$\therefore \quad \phi = \frac{hc}{\lambda} - \frac{R_{max}^2 q^2 B^2}{2m} = 1.0899 \text{ eV} \approx 1.1\text{eV}$$

12. The aperture diameter of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to :

(1) 20 m (2) 600 m

(3) 60 m (4) 200 m

NTA Ans. (3)

Sol. Let distance is x then

$$d\theta = \frac{1.22\lambda}{D} \quad (D = \text{diameter})$$

$$\frac{x}{d} = \frac{1.22\lambda}{D} \text{ (d = distance between earth \& moon)}$$

$$x = \frac{1.22 \times (5500 \times 10^{-10}) \times (4 \times 10^8)}{5} = 53.68\text{m}$$

most appropriate is 60m.

13. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(1) $\frac{3}{4}mu^2$ (2) $\frac{1}{8}mu^2$ (3) $\sqrt{\frac{2}{3}}mu^2$ (4) $\frac{1}{3}mu^2$

NTA Ans. **(2)**

Sol. From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m + m)\bar{v}$$

$$\Rightarrow \quad \bar{v} = \frac{3}{4}u\hat{i} + \frac{u}{4}\hat{j}$$

$$\Rightarrow \quad |v| = \frac{u}{4}\sqrt{10}$$

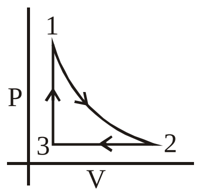
Final kinetic energy $= \frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2 = \frac{5}{8}mu^2$

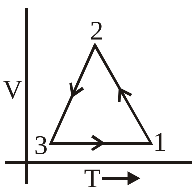
Initial kinetic energy

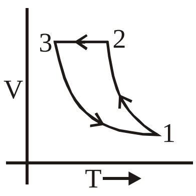
$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

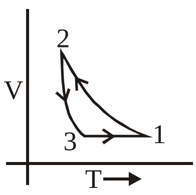
Loss in K.E. $= k_i - k_f = \frac{1}{8}mu^2$

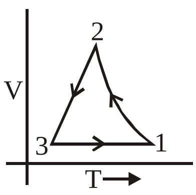
14. Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in the figure ? where, $1 \rightarrow 2$ is adiabatic.
(Graphs are schematic and are not to scale)



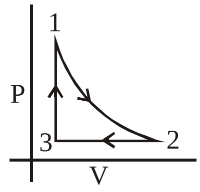
(1)


(2)


(3)


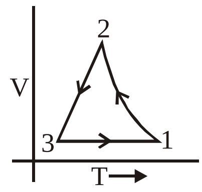
(4)


NTA Ans. **(4)**

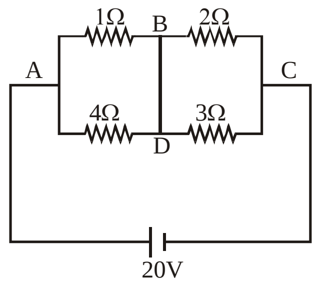
Sol.


In process 2 to 3 pressure is constant & in process 3 to 1 volume is constant which is correct only in option 4.

Correct graph is

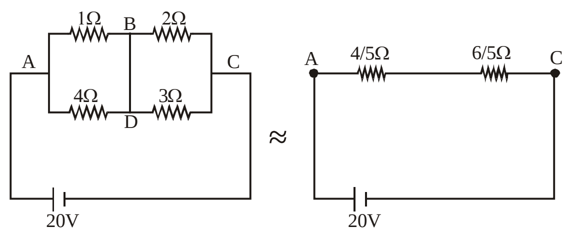


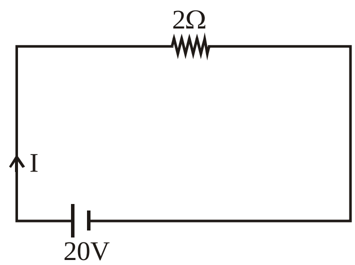
15. In the given circuit diagram, a wire is joining points B and D. The current in this wire is :



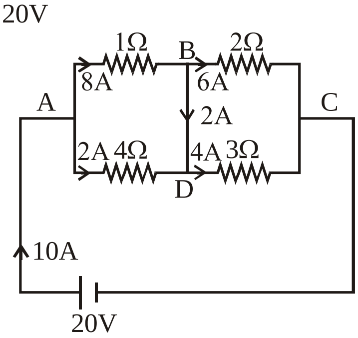
(1) 4A (2) 2A (3) 0.4A (4) Zero

NTA Ans. **(2)**

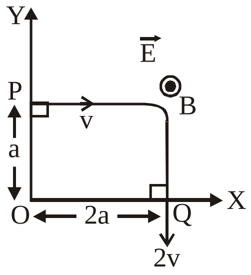
Sol.


$\approx$


$$\Rightarrow I = \frac{20}{2} = 10A$$

Now,


16. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\vec{i}$ and a uniform magnetic field $B\vec{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A, B, C, D) are the correct ? (Trajectory shown is schematic and not to scale) :



(A) $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2 mav.

(1) (A), (B), (C), (D) (2) (A), (B), (C)

(3) (B), (C), (D) (4) (A), (C), (D)

NTA Ans. (2)

Sol. Option (A)

$W = k_f - k_i$

$qE(2a - 0) = \frac{1}{2}m(2V)^2 - \frac{1}{2}mV^2$

$qE2a = \frac{3}{2}mV^2$

$E = \frac{3}{4}\frac{mv^2}{qa}$

Option (B)

Rate of work done $P = \vec{F}.\vec{V} = FV\cos\theta = FV$

Power = qEV

Power = $q\left(\frac{3}{4}\frac{mV^2}{qa}\right)V$

Power = $q\frac{3}{4}\frac{mV^3}{qa}$

Power = $\frac{3}{4}\frac{mV^3}{a}$

Option (C)

Angle between electric force and velocity is 90°, hence rate of work done will be zero at Q.

Option (D)

Initial angular momentum L_i = mVa

Final angular momentum L_f = m(2V) (2a)

Change in angular momentum $L_f - L_i$ = 3mVa

(Note : angular momentum is calculated about O)

17. Three harmonic waves having equal frequency ν and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to :

(1) 5.8 I_0 (2) 0.2 I_0

(3) I_0 (4) 3 I_0

NTA Ans. (1)

Sol. Let amplitude of each wave is A.

Resultant wave equation

$= A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$

$= A\sin\omega t + \sqrt{2}\,A\sin\omega t$

$= \left(\sqrt{2}+1\right)A\sin\omega t$

Resultant wave amplitude = $\left(\sqrt{2}+1\right)A$

as $I \propto A^2$

so $\frac{I}{I_0} = \left(\sqrt{2}+1\right)^2$

$I = 5.8\ I_0$

18. An electric dipole of moment $\vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C .m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k}$ (note that $\vec{r}.\vec{p} = 0$) is parallel to:

(1) $(-\hat{i} + 3\hat{j} - 2\hat{k})$ (2) $(+\hat{i} - 3\hat{j} - 2\hat{k})$

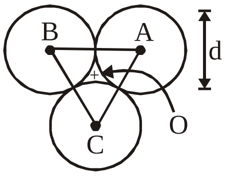
(3) $(+\hat{i} + 3\hat{j} - 2\hat{k})$ (4) $(-\hat{i} - 3\hat{j} + 2\hat{k})$

NTA Ans. (3)

Sol. Since $\vec{r}$ and $\vec{p}$ are perpendicular to each other therefore point lies on the equitorial plane. Therefore electric field at the point will be antiparallel to the dipole moment.

i.e. $\vec{E} \parallel -\vec{p}$

$\vec{E} \parallel (\hat{i} + 3\hat{j} - 2\hat{k})$

19. 

Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is :

(1) $\frac{13}{23}$ (2) $\frac{15}{13}$ (3) $\frac{23}{13}$ (4) $\frac{13}{15}$

NTA Ans. (1)

Sol. From parallel axis theorem

$$I_0 = 3 \times \left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

$$I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2$$

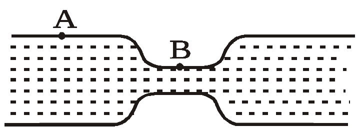
$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2$$

$$\frac{I_0}{I_A} = \frac{13}{23}$$

20. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube.

(density of water = 1000 kgm^{-3})



(Fig.)

(1) 1810 cm^3/s (2) 3020 cm^3/s

(3) 2720 cm^3/s (4) 2420 cm^3/s

NTA Ans. (3)

Sol. Rate of flow of water = $A_AV_A = A_BV_B$

$(40)V_A = (20)V_B$

$V_B = 2V_A$ .... (1)

Using Bernoulli's theorem

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$$

$$700 = \frac{1}{2}\times 1000(4V_A^2 - V_A^2)$$

V_A = 0.68 m/s = 68 cm/s

Rate of flow = A_AV_A

= (40) (68) = 2720 cm^3/s

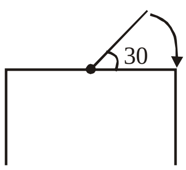
21. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be __________ .

NTA Ans. (10.00)

Sol. $$V = \left|L\frac{di}{dt}\right|$$

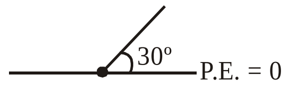
$$\Rightarrow L = \frac{V}{\left|\frac{di}{dt}\right|} = \frac{100}{\frac{0.25}{0.025\times 10^{-3}}} = 10\,mH$$

22. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\,s^{-1}$, where n is an integer. The value of n is __________ .



NTA Ans. (15.00)

Sol.



From mechanical energy conservation,

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow mg\frac{\ell}{2}\sin 30° + 0 = 0 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mg \times \frac{1}{2} \times \frac{1}{2} + 0 = 0 + \frac{1}{2} \times \frac{m(1)^2}{3}\omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{2} \Rightarrow \omega = \sqrt{15}$$

$\therefore$ n = 15

23. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is __________ .

NTA Ans. (3.00)

Sol. $$x = \sqrt{at^2 + 2bt + c}$$

Differentiating w.r.t. time

$$\frac{dx}{dt} = v = \frac{1}{2\sqrt{at^2 + 2bt + c}} \times (2at + 2b)$$

$$\Rightarrow v = \frac{at + b}{x}$$

$\Rightarrow$ vx = at + b

Differentiating w.r.t. x

$$\Rightarrow \frac{dv}{dx} \times x + v = a \times \frac{dt}{dx}$$

Multiply both side by v

$$\Rightarrow \left(v\frac{dv}{dx}\right)x + v^2 = a$$

$$\Rightarrow a'x = a - v^2 \quad \text{[Here a' is acceleration]}$$

$$\Rightarrow a'x = a - \left(\frac{at+b}{x}\right)^2$$

$$\Rightarrow a'x = \frac{ax^2 - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{a(at^2 + 2bt + c) - (at+b)^2}{x^2}$$

$$\Rightarrow a'x = \frac{ac - b^2}{x^2}$$

$$\Rightarrow a' = \frac{ac - b^2}{x^3}$$

$$\therefore a' \propto \frac{1}{x^3} \qquad \therefore n = 3$$

24. A body of mass m = 10 kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire = 4.8×10^7 Nm^{-2} and area of cross-section of the wire = 10^{-2} cm^2) is:

NTA Ans. (4.00)

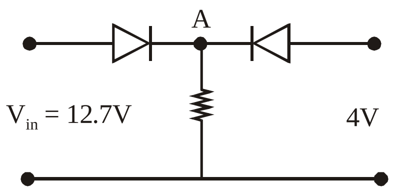
Sol. $T = m\omega^2\ell$

Breaking stress = $\frac{T}{A} = \frac{m\omega^2\ell}{A}$

$$\Rightarrow \omega^2 = \frac{4.8 \times 10^7 \times (10^{-2} \times 10^{-4})}{10 \times 0.3} = 16$$

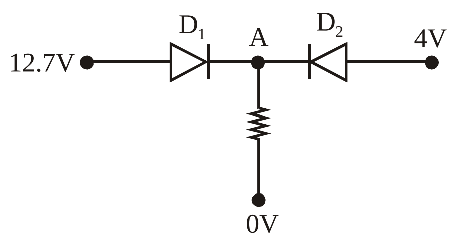
$$\Rightarrow \omega = 4$$

25. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is __________ .



NTA Ans. (12.00)

Sol.



Diode D_1 is forward biased and D_2 is reverse biased.

$\therefore$ V_A = 12.7 – 0.7 = 12V.

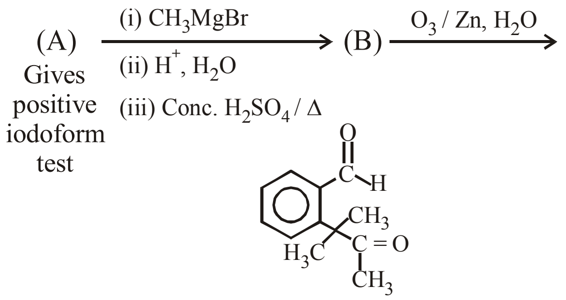
FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

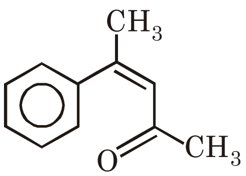
(Held On Thursday 09th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

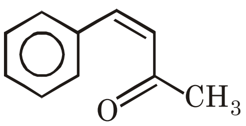
CHEMISTRY

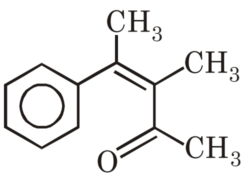
TEST PAPER WITH ANSWER & SOLUTION

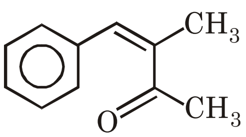
1. Identify (A) in the following reaction sequence :



(1) 

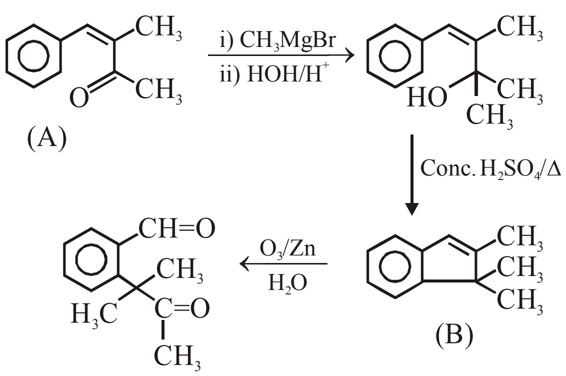
(2) 

(3) 

(4) 

NTA Ans. (4)

Sol.



2. For the following reactions

$A \xrightarrow{700\,K} \text{Product}$

$A \xrightarrow[\text{catalyst}]{500\,K} \text{Product}$

it was found that E_a is decreased by 30 kJ/mol in the presence of catalyst.

If the rate remains unchanged, the activation energy for catalysed reaction is (Assume pre exponential factor is same):

(1) 135 kJ/mol (2) 105 kJ/mol

(3) 198 kJ/mol (4) 75 kJ/mol

NTA Ans. (4)

Sol. $K_1 = Ae^{-\frac{Ea}{R\times 700}}$

$K_2 = A \times e^{-\frac{(Ea-30)}{R\times 500}}$

For same rate

$K_1 = K_2$

$e^{-\frac{Ea}{700R}} = e^{-\frac{(Ea-30)}{R\times 500}}$

$$\frac{Ea}{700R} = \frac{Ea-30}{R\times 500}$$

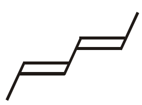
$5Ea = 7Ea - 210$

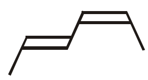
$210 = 2Ea$

$E_a = 105$ kJ/mole

$E_a - 30 = 75$

3. The correct order of heat of combustion for following alkadienes is :

(a) 

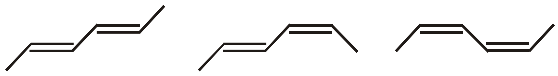
(b) 

(c) 

(1) (a) < (b) < (c) (2) (b) < (c) < (a)

(3) (c) < (b) < (a) (4) (a) < (c) < (b)

NTA Ans. (1)

Sol. 

(a) (b) (c)

(Trans, Trans) (Trans, Cis) (Cis, Cis)

$\therefore$ Generally trans is more stable then cis form.

Heat of combustion (HOC) $\propto \dfrac{1}{\text{Stability}}$

Stability : a > b > c

HOC : c > b > a

4. A chemist has 4 samples of artificial sweetener A, B, C and D. To identify these samples, he performed certain experiments and noted the following observations :

(i) A and D both form blue-violet colour with ninhydrin.

(ii) Lassaigne extract of C gives positive $AgNO_3$ test and negative $Fe_4[Fe(CN)_6]_3$ test.

(iii) Lassaigne extract of B and D gives positive sodium nitroprusside test

Based on these observations which option is correct ?

(1) A : Aspartame ; B : Saccharin ; C : Sucralose ; D ; Alitame

(2) A : Alitame ; B : Saccharin ; C : Aspartame ; D ; Sucralose

(3) A : Saccharin ; B : Alitame ; C : Sucralose ; D ; Aspartame

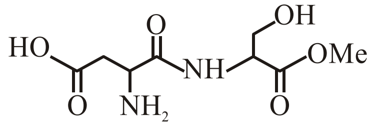
(4) A : Aspartame ; B : Alitame ; C : Saccharin ; D ; Sucralose

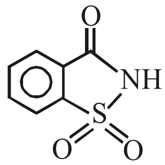
NTA Ans. (1)

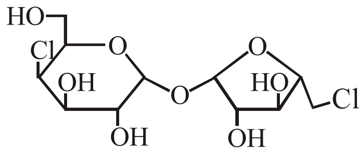
Sol. (i) Blue voilet color with Ninhydrine $\rightarrow$ amino acid derivative. So it cannot be saccharide or sucralose.

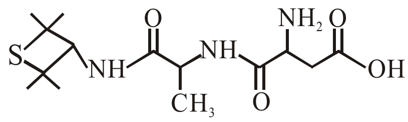
(ii) Lassaigne extract give +ve test with $AgNO_3$. So Cl is present, –ve test with $Fe_4[Fe(CN)_6]_3$ means N is absent. So it can't be Aspartame or Saccharine or Alitame, so C is sucralose.

(iii) Lassaigne solution of B and D given +ve sodium nitroprusside test, so it is having S, so it is Saccharine and Alitame.

(A) Aspartame 

(B) Saccharine 

(C) Sucralose 

(D) Alitame 

5. 'X' melts at low temperature and is a bad conductor of electricity in both liquid and solid state. X is :

(1) Carbon tetrachloride

(2) Mercury

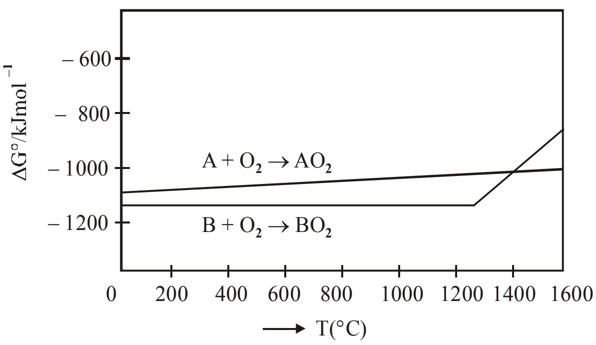
(3) Silicon carbide

(4) Zinc sulphide

NTA Ans. (1)

Sol. CCl_4 is molecular solid so does not conduct electricity in liquid & solid state.

6. According to the following diagram, A reduces BO_2 when the temperature is :



(1) < 1400 °C

(2) > 1400 °C

(3) < 1200 °C

(4) > 1200 °C but < 1400 °C

NTA Ans. (2)

Sol. A reduces BO_2 when temperature is above 1400°C because above 1400°C A has more –ve ΔG° for AO_2 formation than B to BO_2 formation.

7. The K_{sp} for the following dissociation is 1.6×10^{-5}

$$PbCl_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2Cl^-_{(aq)}$$

Which of the following choices is correct for a mixture of 300 mL 0.134 M $Pb(NO_3)_2$ and 100 mL 0.4 M NaCl ?

(1) $Q < K_{sp}$

(2) $Q > K_{sp}$

(3) $Q = K_{sp}$

(4) Not enough data provided

NTA Ans. (2)

Sol. $\left[Pb^{2+}\right] = \dfrac{300 \times 0.134}{400}$

$= 1.005 \times 10^{-1}$ M

$\left[Cl^-\right] = \dfrac{100 \times 0.4}{400}$

$= 10^{-1}$ M

$$PbCl_{2(s)} \rightleftharpoons Pb^{+2}_{(aq.)} + 2Cl^-_{(aq.)}$$

$Q = [Pb^{2+}] \times [Cl^-]^2$

$= 1.005 \times 10^{-3} > k_{sp}$

8. $[Pd(F)(Cl)(Br)(I)]^{2-}$ has n number of geometrical isomers. Then, the spin-only magnetic moment and crystal field stabilisation energy [CFSE] of $[Fe(CN)_6]^{n-6}$, respectively, are:

[Note : Ignore the pairing energy]

(1) 2.84 BM and $-1.6\ \Delta_0$

(2) 1.73 BM and $-2.0\ \Delta_0$

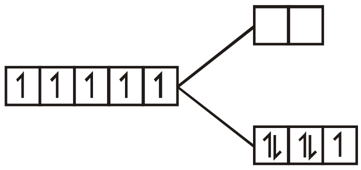
(3) 0 BM and $-2.4\ \Delta_0$

(4) 5.92 BM and 0

NTA Ans. (2)

Sol. $[Pb(F)(Cl)(Br)(I)]^{2-}$ have three geometrical isomer so formula for $[Fe(CN)_6]^{n-6}$ is $[Fe(CN)_6]^{3-}$ and CFSE for this complex is

$Fe^{3\oplus} \Rightarrow 3d^5 4s^\circ$



Magnetic Moment $= \sqrt{3}$

$= 1.73$ B.M

CFSE $= [(-0.4\times5) + (0.6 \times 0)]\ \Delta_0$

$= -2.0\ \Delta_0$

9. If the magnetic moment of a dioxygen species is 1.73 B.M, it may be :

(1) O_2^- or O_2^+

(2) O_2 or O_2^+

(3) O_2 or O_2^-

(4) O_2, O_2^- or O_2^+

NTA Ans. (1)

Sol.

	number of unpaired electron	magnetic moment
$O_2^\ominus$	1	1.73 B.M
$O_2^\oplus$	1	1.73 B.M
O_2	2	2.83 BM

10. If enthalpy of atomisation for $Br_{2(l)}$ is x kJ/mol and bond enthalpy for Br_2 is y kJ/mol, the relation between them :

(1) is x = y (2) is x < y

(3) does not exist (4) is x > y

NTA Ans. (4)

Sol. Enthalpy of atomisation of $Br_2(l)$

$$Br_2(l) \xrightarrow{\Delta H_{vap}} Br_2(g) \xrightarrow{\Delta H_{BE}} 2Br(g)$$

$\Delta H_{atom.}$

$\Delta H_{atom} = \Delta H_{vap} + \Delta H_{BE}$

$x = \Delta H_{vap} + y$

So, $x > y$

11. The increasing order of basicity for the following intermediates is (from weak to strong)

(i) $H_3C-\overset{\ominus}{C}(CH_3)-CH_3$

(ii) $H_2C=CH-\overset{\ominus}{C}H_2$

(iii) $HC\equiv\overset{\ominus}{C}$ (iv) $\overset{\ominus}{C}H_3$ (v) $\overset{\ominus}{C}N$

(1) (v) < (i) < (iv) < (ii) < (iii)

(2) (iii) < (i) < (ii) < (iv) < (v)

(3) (v) < (iii) < (ii) < (iv) < (i)

(4) (iii) < (iv) < (ii) < (i) < (v)

NTA Ans. (3)

Sol. $CH_3-\overset{\ominus}{C}(CH_3)-CH_3$ (i) $CH_2=CH-CH_2^{\ominus}$ (ii) $HC\equiv C^{\ominus}$ (iii)

$CH_3^{\ominus}$ (iv) $\overset{\ominus}{C}\equiv N$ (v)

Basic strength order : (i) > (iv) > (ii) > (iii) > (v)

12. B has a smaller first ionization enthalpy than Be. Consider the following statements :

(I) It is easier to remove 2p electron than 2s electron

(II) 2p electron of B is more shielded from the nucleus by the inner core of electrons than the 2s electrons of Be.

(III) 2s electron has more penetration power than 2p electron.

(IV) atomic radius of B is more than Be

(Atomic number B = 5, Be = 4)

The correct statements are :

(1) (I), (II) and (III)

(2) (II), (III) and (IV)

(3) (I), (III) and (IV)

(4) (I), (II) and (IV)

NTA Ans. (1)

Sol. Be ⇒ $1s^2\ 2s^2$

B ⇒ $1s^2\ 2s^2\ 2p^1$

B has a smaller size than Be

it is easier to remove 2p electron than 2s electron due to less pentration effect of 2p than 2s.

2p electron of Boron is more shielded from the nucleus by the inner core of electron than the 2s electron of Be

B has a smaller size than Be

13. The acidic, basic and amphoteric oxides, respectively, are :

(1) MgO, Cl_2O, Al_2O_3

(2) Cl_2O, CaO, P_4O_{10}

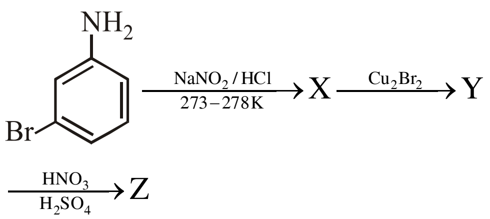
(3) Na_2O, SO_3, Al_2O_3

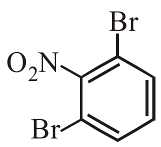
(4) N_2O_3, Li_2O, Al_2O_3

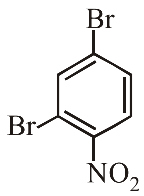
NTA Ans. (4)

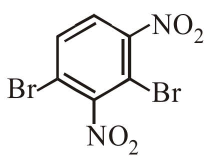
Sol. 1. MgO Basic
Cl_2O Acidic
Al_2O_3 amphoteric
2. Cl_2O Acidic
CaO Basic
P_4O_{10} Acidic
3. Na_2O Basic
SO_3 Acidic
Al_2O_3 amphoteric
4. N_2O_3 Acidic
Li_2O Basic
Al_2O_3 amphoteric

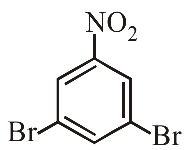
14. The major product Z obtained in the following reaction scheme is :



(1) 

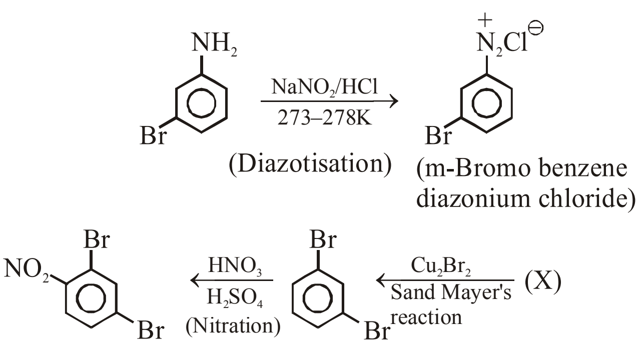
(2) 

(3) 

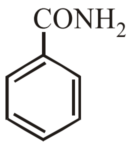
(4) 

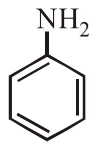
NTA Ans. (2)

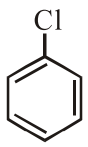
Sol.

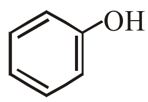


15. Which of these will produce the highest yield in Friedel Crafts reaction?

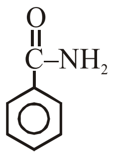
(1) 

(2) 

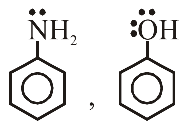
(3) 

(4) 

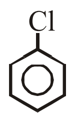
NTA Ans. (3)

Sol. ∴ 

(Deactivated ring due to –R effect of amide)



(l.p.e. of $-NH_2$ and $-\ddot{O}H$ group do acid-base reaction with lewis acid $AlCl_3$, so ring is deactivated)



(Highest yield produced)

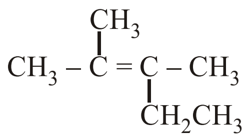
16. The major product (Y) in the following reactions is :

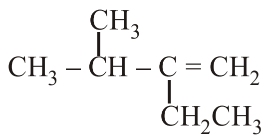
$$CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-C\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} X$$

$$\xrightarrow[\text{(ii) Conc. } H_2SO_4/\Delta]{\text{(i) } C_2H_5MgBr, H_2O} Y$$

(1) $H_3C-\overset{CH_2}{\overset{\|}{C}}-\underset{C_2H_5}{\underset{|}{C}H}-CH_3$

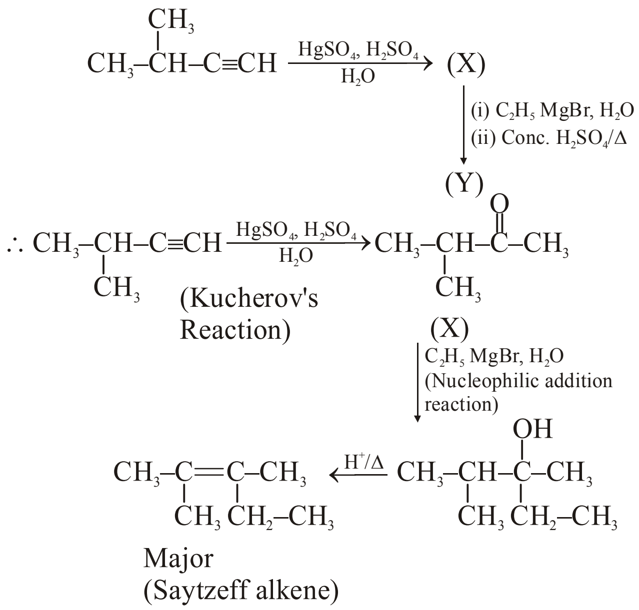
(2) $CH_3-\overset{CH_3}{\overset{|}{C}H}-\underset{CH_3}{\underset{|}{C}}=CH-CH_3$

(3) 

(4) 

NTA Ans. (3)

Sol.



17. Complex X of composition $Cr(H_2O)_6Cl_n$ has a spin only magnetic moment of 3.83 BM. It reacts with $AgNO_3$ and shows geometrical isomerism. The IUPAC nomenclature of X is :

(1) Tetraaquadichlorido chromium (III) chloride dihydrate

(2) Hexaaqua chromium (III) chloride

(3) Dichloridotetraaqua chromium (IV) chloride dihydrate

(4) Tetraaquadichlorido chromium(IV) chloride dihydrate

NTA Ans. (1)

Sol. $Cr(H_2O)_6\ Cl_n$

if magnetic mement is 3.83 BM then it contain three unpaired electrons. It means chromium in +3 oxidation state so molecular formula is $Cr(H_2O)_6\ Cl_3$

∴ This formula have following isomers

(a) $[Cr(H_2O)_6]Cl_3$: react with $AgNO_3$ but does not show geometrical isomerism.

(b) $[Cr(H_2O)_5Cl]Cl_2.H_2O$ react with $AgNO_3$ but does not show geometrical isomerism.

(c) $[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism.

(d) $[Cr(H_2O)_3Cl_3].3H_2O$ does not react with $AgNO_3$ & show geometrical isomerism.

$[Cr(H_2O)_4Cl_2]Cl.2H_2O$ react with $AgNO_3$ & show geometrical isomerism and it's IUPAC nomenclature is Tetraaquadichlorido chromium (III) Chloride dihydrate.

18. The compound that cannot act both as oxidising and reducing agent is :

(1) H_2O_2 (2) H_2SO_3

(3) HNO_2 (4) H_3PO_4

NTA Ans. (4)

Sol. (i) H_2O_2 act as oxidising agent as well as reducing agent depending on condition.

(ii) H_2SO_3 act as oxidising agent as well as reducing agent depending on condition.

(iii) HNO_2 act as oxidising agent as well as reducing agent depending on condition.

(iv) H_3PO_4 can not act both as oxidising and reducing agent.

H_3PO_4 can act as only oxidising agent.

$$H_3PO_4 \rightleftharpoons 3H^+ + PO_4^{3-}$$

19. The de Broglie wavelength of an electron in the 4^{th} Bohr orbit is :

(1) $8\pi a_0$

(2) $2\pi a_0$

(3) $4\pi a_0$

(4) $6\pi a_0$

NTA Ans. (1)

Sol. $2\pi r = n\lambda$

for $n = 1, r = a_0$

$n = 4, r = 16a_0$

So, $2\pi \times 16a_0 = 4 \times \lambda$

$\lambda = 8\pi a_0$

20. The electronic configurations of bivalent europium and trivalent cerium are

(atomic number : Xe = 54, Ce = 58, Eu = 63)

(1) [Xe] $4f^4$ and [Xe] $4f^9$

(2) [Xe] $4f^7$ and [Xe] $4f^1$

(3) [Xe] $4f^7$ $6s^2$ and [Xe] $4f^2$ $6s^2$

(4) [Xe] $4f^2$ and [Xe] $4f^7$

NTA Ans. (2)

Sol. $Eu_{63} \Rightarrow [Xe]\ 4f^7\ 5d^{\circ}\ 6s^2$

$Eu^{2\oplus} \Rightarrow [Xe]\ 4f^7$

$Ce_{58} \Rightarrow [Xe]\ 4f^1\ 5d^1\ 6s^2$

$Ce^{3\oplus} \Rightarrow [Xe]\ 4f^1$

21. The hardness of a water sample containing 10^{-3} M $MgSO_4$ expressed as $CaCO_3$ equivalents (in ppm) is ______.

(molar mass of $MgSO_4$ is 120.37 g/mol)

NTA Ans. (100 to 100)

Sol. 1 Litre has 10^{-3} moles $MgSO_4$

So , 1000 litre has 1 mole $MgSO_4$

= 1 mole $CaCO_3$

= 100 ppm

22. The molarity of HNO_3 in a sample which has density 1.4 g/mL and mass percentage of 63% is _____. (Molecular Weight of HNO_3 = 63)

NTA Ans. (14.00 to 14.00)

Sol. 100 gm soln $\rightarrow$ 63 gm HNO_3

$\frac{100}{1.4}$ mL $\rightarrow$ 1 mole HNO_3

$$\text{Molarity} = \frac{1}{\frac{100}{1.4} \times \frac{1}{1000}} = 14M$$

23. 108 g of silver (molar mass 108 g mol^{-1}) is deposited at cathode from $AgNO_3$(aq) solution by a certain quantity of electricity. The volume (in L) of oxygen gas produced at 273 K and 1 bar pressure from water by the same quantity of electricity is _______.

NTA Ans. (5.66 to 5.67)

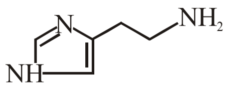
Sol. gm eq. of Ag = $\frac{108}{108}$ = 1

gm eq. of $O_2(g) = 1$

Volume of $O_2(g) = 22.7 \times \frac{1}{4}$ = 5.675 litre

24. The mass percentage of nitrogen in histamine is _____.

NTA Ans. (37.80 to 38.20)

Sol. 

M.F. of Histamine is $C_5H_9N_3$

Molecular mass of Histamine is 111

Now, mass % of nitrogen = $\left(\frac{42}{111}\right)\times 100$

= 37.84%

25. How much amount of NaCl should be added to 600 g of water (ρ = 1.00 g/mL) to decrease the freezing point of water to – 0.2 °C ? ______. (The freezing point depression constant for water = 2K kg mol^{-1})

NTA Ans. (1.74 to 1.76)

Sol. $\Delta T_f = i \times m \times K_f$

$0.2 = 2 \times 2 \times \frac{w/58.5}{600/1000}$

w = 1.755 gm

FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

(Held On Thursday 09th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM

MATHEMATICS

TEST PAPER WITH ANSWER & SOLUTION

1. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is :

(1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

NTA Ans. (3)

Sol. Let thickness of ice be 'h'.

Vol. of ice = $v=\frac{4\pi}{3}\left((10+h)^3-10^3\right)$

$$\frac{dv}{dt}=\frac{4\pi}{3}\left(3(10+h)^2\right).\frac{dh}{dt}$$

Given $\frac{dv}{dt}=50cm^3/min$ and h = 5cm

$\Rightarrow$ $50=\frac{4\pi}{3}\left(3(10+5)^2\right)\frac{dh}{dt}$

$\Rightarrow$ $\frac{dh}{dt}=\frac{50}{4\pi\times15^2}=\frac{1}{18\pi}cm/min$

2. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336 k, then k is equal to :

(1) 8 (2) 6 (3) 4 (4) 7

NTA Ans. (1)

Sol. _ _ _ 2 _

No. of five digits numbers =

No. of ways of filling remaining 4 places

= 8 × 8 × 7 × 6

$\therefore$ $k=\frac{8\times8\times7\times6}{336}=8$

3. Let z be complex number such that $\left|\frac{z-i}{z+2i}\right|=1$ and $|z|=\frac{5}{2}$. Then the value of $|z + 3i|$ is :

(1) $\sqrt{10}$ (2) $2\sqrt{3}$ (3) $\frac{7}{2}$ (4) $\frac{15}{4}$

NTA Ans. (3)

Sol. $\left|\frac{z-i}{z+2i}\right|=1$

$\Rightarrow$ $|z - i| = |z + 2i|$

$\Rightarrow$ z lies on perpendicular bisector of (0, 1) and (0, –2).

$\Rightarrow$ $Imz=-\frac{1}{2}$

Let $z=x-\frac{i}{2}$

$\because$ $|z|=\frac{5}{2}$ $\Rightarrow$ $x^2=6$

$\therefore$ $|z+3i|=\left|x+\frac{5i}{2}\right|=\sqrt{x^2+\frac{25}{4}}$

$=\sqrt{6+\frac{25}{4}}=\frac{7}{2}$

4. In a box, there are 20 cards, out of which 10 are lebelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :

(1) $\frac{11}{16}$ (2) $\frac{13}{16}$ (3) $\frac{9}{16}$ (4) $\frac{15}{16}$

NTA Ans. (1)

Sol. A : Event when card A is drawn

B : Event when card B is drawn.

$$P(A) = P(B) = \frac{1}{2}$$

Required probability = P(AA or (AB)A or (BA)A or (ABB)A or (BAB)A or (BBA)A)

$$= \frac{1}{2}\times\frac{1}{2} + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 2 + \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times 3$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{3}{16} = \frac{11}{16}$$

5. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to :

(1) 2π (2) 4π (3) $2\pi^2$ (4) π^2

NTA Ans. (4)

Sol. $I = \int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x} dx$ ....(1)

$$= \left[\int_0^{\pi} \frac{x\sin^8 x}{\sin^8 x \cos^8 x}dx + \int_0^{\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}dx\right]$$

$$= 2\pi\int_0^{\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x}dx$$

$$I = 2\pi\left[\int_0^{\pi/2} \frac{\sin^8 x}{\sin^8 x + \cos^8 x}dx + \int_0^{\pi/2} \frac{\cos^8 x dx}{\sin^8 x + \cos^8 x}dx\right]$$

$$= 2\pi\int_0^{\pi/2} 1 dx = 2\pi.\frac{\pi}{2} = \pi^2$$

6. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

(1) $\frac{\pi - 1}{4}$ (2) $\frac{\pi + 2}{4}$

(3) $\frac{\pi + 1}{4}$ (4) $\frac{1}{4}$

NTA Ans. (3)

Sol. $f'(x) = \tan^{-1}(\sec x + \tan x)$

$$f'(x) = \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{\pi}{2}\right)\right)$$

$\because\ -\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$

$\Rightarrow\ f'(x) = \frac{\pi}{4} + \frac{x}{2}$

$\therefore\ f(x) = \frac{\pi}{4}.x + \frac{x^2}{4} + c$

$\because\ f(0) = 0 \Rightarrow c = 0$

$\Rightarrow\ f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

$\therefore\ f(1) = \frac{\pi + 1}{4}$

7. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adjA and C = 3A, then $\frac{|adjB|}{|C|}$ is equal to :

(1) 72 (2) 2 (3) 8 (4) 16

NTA Ans. (3)

Sol. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

$\Rightarrow\ |A| = 6$

$$\frac{|adjB|}{|c|} = \frac{|adj(adjA)|}{|9A|} = \frac{|A|^4}{3^3|A|} = \frac{|A|^3}{3^3}$$

$$= \frac{(6)^3}{(3)^3} = 8$$

8. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is :

(1) 4 (2) 2 (3) 3 (4) 1

NTA Ans. (4)

Sol. $e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$$\Rightarrow \quad e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0$$

$$\Rightarrow \quad \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

$$\Rightarrow \quad \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

9. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is irrational or 5 is an integer.

(2) $\sqrt{5}$ is not an integer and 5 is not irrational.

(3) $\sqrt{5}$ is an integer and 5 is irrational.

(4) $\sqrt{5}$ is not an integer or 5 is not irrational.

NTA Ans. (2)

Sol. $p = \sqrt{5}$ is an integer.

q : 5 is irrational

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

= $\sqrt{5}$ is not an integer and 5 is not irrational.

10. Let the observations $x_i (1 \le i \le 10)$ satisfy the equations, $\sum_{i=1}^{10}(x_i - 5) = 10$ and $\sum_{i=1}^{10}(x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, \ldots, x_{10} - 3$, then the ordered pair (μ, λ) is equal to :

(1) (6, 6) (2) (3, 6)

(3) (6, 3) (4) (3, 3)

NTA Ans. (4)

Sol. $\sum_{i=1}^{10}(x_i - 5) = 10$

$\Rightarrow$ Mean of observation $x_i - 5 = \frac{1}{10}\sum_{i=1}^{3}(x_i - 5) = 1$

$\Rightarrow \mu$ = mean of observation $(x_i - 3)$

= (mean of observation $(x_i - 5)$) + 2

= 1 + 2 = 3

Variance of observation

$x_i - 5 = \frac{1}{10}\sum_{i=1}^{10}(x_i - 5)^2 - (\text{Mean of } (x_i - 5))^2 = 3$

$\Rightarrow \lambda$ = variance of observation $(x_i - 3)$

= variance of observation $(x_i - 5) = 3$

$\therefore (\mu, \lambda) = (3, 3)$

11. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots$ to ∞ is equal to :

(1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$ (3) 2 (4) 1

NTA Ans. (1)

Sol. $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots \infty$

$= 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdot \ldots \infty$

$= 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdot \ldots \infty$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$

12. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle ?

(1) $3x - 4y - 24 = 0$ (2) $3x + 4y - 6 = 0$

(3) $4x + 3y - 8 = 0$ (4) $4x - 3y + 17 = 0$

NTA Ans. (3)

Sol. Equation of family of circle touching y-axis at (0, 4) is given by $(x - 0)^2 + (y - 4)^2 + \lambda x = 0$.

$\because$ It passes through (2, 0)

$\Rightarrow \lambda = -10.$

$\Rightarrow$ Required circle is $(x - 0)^2 + (y - 4)^2 - 10x = 0$

$\Rightarrow x^2 + y^2 - 10x - 8y + 16 = 0$

center of circle $\equiv (5, 4)$ and radius = 5

distance of $4x + 3y - 8 = 0$ from (5, 4)

$= \left|\frac{24}{5}\right| \neq$ radius

13. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola, $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

(1) 15 (2) 14 (3) 17 (4) 16

NTA Ans. (4)

Sol. $e_1=\sqrt{1-\frac{4}{18}}=\frac{\sqrt{7}}{3}$

$e_2=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$

$\because$ (e_1, e_2) lies on $15x^2 + 3y^2 = k$

$\Rightarrow$ $15e_1^2+3e_2^2=k$

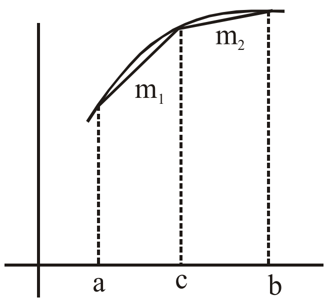
$\Rightarrow$ $k = 16$

14. Let f be any function continuous on [a, b] and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\frac{f(c)-f(a)}{f(b)-f(c)}$ is greater than :

(1) $\frac{b+a}{b-a}$ (2) $\frac{b-c}{c-a}$ (3) $\frac{c-a}{b-c}$ (4) 1

NTA Ans. (3)

Sol.



it is clear from graph that $m_1 > m_2$

$\Rightarrow$ $\frac{f(c)-f(a)}{c-a}>\frac{f(b)-f(c)}{b-c}$

$\Rightarrow$ $\frac{f(c)-f(a)}{f(b)-f(c)}>\frac{c-a}{b-c}$

15. If for some α and β in R, the intersection of the following three places

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to :

(1) 10 (2) –10 (3) 2 (4) 0

NTA Ans. (1)

Sol. For planes to intersect on a line

$\Rightarrow$ there should be infinite solution of the given system of equations

for infinite solutions

$$\Delta=\begin{vmatrix}1 & 4 & -2\\ 1 & 7 & -5\\ 1 & 5 & \alpha\end{vmatrix}=0 \Rightarrow 3\alpha + 9 = 0 \Rightarrow \alpha = -3$$

$$\Delta_z=\begin{vmatrix}1 & 4 & 1\\ 1 & 7 & \beta\\ 1 & 5 & 5\end{vmatrix}=0\Rightarrow 13-\beta=0\Rightarrow\beta=13$$

Also for $\alpha = -3$ and $b = 13$ $\Delta_x = \Delta_y = 0$

$\therefore$ $\alpha + \beta = -3 + 13 = 10$

16. The integral $\int\frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$

(2) $-\left(\frac{x-3}{x+4}\right)^{-\frac{1}{7}}+C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}}+C$

(4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-\frac{13}{7}}+C$

NTA Ans. (1)

Sol. $I = \int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}} = \int \frac{dx}{\left(\frac{x+4}{x-3}\right)^{\frac{8}{7}}(x-3)^2}$

Let $\frac{x+4}{x-3} = t \Rightarrow \frac{dx}{(x-3)^2} = -\frac{1}{7}dt$

$\Rightarrow \quad I = -\frac{1}{7}\int \frac{dt}{t^{8/7}} = -\frac{1}{7}\int t^{-8/7}dt$

$= t^{-1/7} + C = +\left(\frac{x+4}{x-3}\right)^{-1/7} + C = \left(\frac{x-3}{x+4}\right)^{1/7} + C$

17. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines x + 3y – 1 = 0 and 3x – y + 1 = 0. Then the line passing through the points C and P also passes through the point :

(1) (7, 6) (2) (–9, –6)

(3) (–9, –7) (4) (9, 7)

NTA Ans. (2)

Sol. Centroid of Δ = (2, 2)

line passing through intersection of

x + 3y – 1 = 0 and

3x – y + 1 = 0, be given by

(x + 3y – 1) + λ(3x – y + 1) = 0

∵ It passes through (2, 2)

$\Rightarrow \quad 7 + 5\lambda = 0 \Rightarrow \lambda = -\frac{7}{5}$

∴ Required line is 8x – 11y + 6 = 0

∵ (–9, –6) satisfies this equation.

18. If $f(x) = \begin{cases} \frac{\sin(a+2)x + \sin x}{x} & ; x < 0 \\ b & ; x = 0 \\ \frac{(x+3x^2)^{\frac{1}{3}} - x^{\frac{1}{3}}}{x^{\frac{4}{3}}} & ; x > 0 \end{cases}$

is continuous at x = 0, then a + 2b is equal to :

(1) –1 (2) 1 (3) –2 (4) 0

NTA Ans. (4)

Sol. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\left(\frac{\sin(a+2)x}{x} + \frac{\sin x}{x}\right) = a+3$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}}$

$= \lim_{x\to 0}\frac{(1+3x)^{1/3} - 1}{x} = 1$

$f(0)$ = b

for continuity at x = 0

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

⇒ a + 3 = b = 1

∴ a = –2, b = 1

∴ a + 2b = 0

19. The value of

$\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right)$

is :

(1) $\frac{1}{4}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{2}$

NTA Ans. (3)

Sol. $\cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8} + \sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$

$= \sin\frac{\pi}{8}.\cos\frac{\pi}{8} = \frac{1}{2}\sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

20. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$;

then $\int_0^1 f(x)dx$ is equal to :

(1) $\frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\}$

(2) $2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\}$

NTA Ans. (3)

Sol. $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)dx = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$$

$$= a + \frac{b}{2} + \frac{c}{3} = \frac{1}{6}[6a + 3b + c]$$

$$= \frac{1}{6}\left[f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right]$$

21. The coefficient of x^4 is the expansion of $(1 + x + x^2)^{10}$ is ______.

NTA Ans. (615.00)

Sol. $(1 + x + x^2)^{10}$

$= {}^{10}C_0 + {}^{10}C_1 x(1 + x) + {}^{10}C_2 x^2(1 + x)^2$

$+ {}^{10}C_3 x^3(1 + x)^3 + {}^{10}C_4 x^4(1 + x)^4 + ...$

Coeff. of $x^4 = {}^{10}C_2 + {}^{10}C_3 \times {}^3C_1 + {}^{10}C_4 = 615.$

22. The number of distinct solutions of the equation $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is ______.

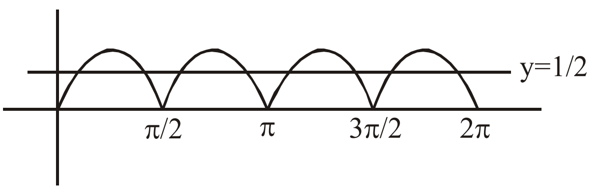
NTA Ans. (8.00)

Sol. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$; $x \in [0, 2\pi]$

$\Rightarrow$ $\log_{1/2}|\sin x| + \log_{1/2}|\cos x| = 2$

$\Rightarrow$ $\log_{1/2}(|\sin x \cos x|) = 2$

$\Rightarrow$ $|\sin x \cos x| = \frac{1}{4}$ $\Rightarrow$ $|\sin 2x| = \frac{1}{2}$



$\Rightarrow$ 8 solutions

23. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation

$(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$,

then $y(3)$ is equal to ______.

NTA Ans. (3.00)

Sol. $(x + 1)dy - ydx = ((x + 1)^2 - 3)dx$

$$\Rightarrow \frac{(x+1)dy - ydx}{(x+1)^2} = \left(1 - \frac{3}{(x+1)^2}\right)dx$$

$$\Rightarrow d\left(\frac{y}{(x+1)}\right) = \left(1 - \frac{3}{(x+1)^2}\right)dx$$

integrating both sides

$$\frac{y}{x+1} = x + \frac{3}{(x+1)} + C$$

Given $y(2) = 0$ $\Rightarrow$ $c = -3$

$$\therefore \quad y = (x+1)\left(x + \frac{3}{(x+1)} - 3\right)$$

$$\therefore \quad y(3) = 3.00$$

24. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,

$\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and

$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in R)$ are coplanar

and $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is ______.

NTA Ans. (1.00)

Sol. $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$,

$\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and

$\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$

$\because$ $\vec{p}, \vec{q}, \vec{r}$ are coplanar

$\Rightarrow$ $[\vec{p}\ \ \vec{q}\ \ \vec{r}] = 0$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow$ $3a + 1 = 0$ $\Rightarrow$ $a = -\frac{1}{3}$

$\vec{p}.\vec{q} = -\frac{1}{3}$, $\vec{r}.\vec{q} = -\frac{1}{3}$

$|\vec{r}|^2 = |\vec{q}|^2 = \frac{2}{3}$

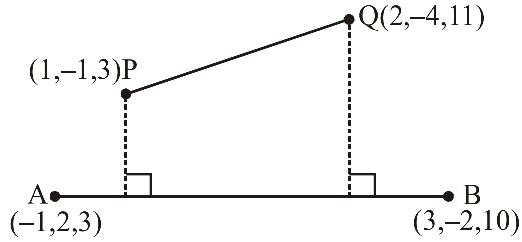
$\therefore$ $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$

$$\Rightarrow \lambda = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r} \times \vec{q}|^2} = \frac{3(\vec{p}.\vec{q})^2}{|\vec{r}|^2|\vec{q}|^2 - (\vec{r}.\vec{q})^2} = 1.00$$

25. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ______.

NTA Ans. (8.00)

Sol.



Projection of $\overrightarrow{PQ}$ on $\overrightarrow{AB} = \left|\frac{\overrightarrow{PQ} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|}\right|$

$$= \left|\frac{(\hat{i} - 3\hat{j} + 8\hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k})}{9}\right| = 8$$