# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM**

## PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

**1.** Consider a solid sphere of radius R and mass density $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$, $0<r\le R$. The minimum density of a liquid in which it will float is :

(1) $\frac{\rho_0}{5}$ (2) $\frac{\rho_0}{3}$

(3) $\frac{2\rho_0}{3}$ (4) $\frac{2\rho_0}{5}$

**NTA Ans. (4)**

**Sol.** In case of minimum density of liqued, sphere will be floating while completely submerged

So mg = B

$$m=\int_0^R \rho(4\pi r^2 dr)=B$$

$$=\rho_0\int_0^R\left(1-\frac{r^2}{R^2}\right)4\pi r^2 dr=\frac{4}{3}\pi R^3\rho_\ell g$$

On Solving

$$\rho_\ell=\frac{2\rho_0}{5}$$

**2.** When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy $T_A$ eV end de-Broglie wavelength $\lambda_A$. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is :

(1) 3eV (2) 2eV

(3) 4eV (4) 1.5eV

**NTA Ans. (3)**

**Sol.** $\lambda_B = 2\lambda_A$

$$\Rightarrow \frac{h}{\sqrt{2T_B m}}=\frac{2h}{\sqrt{2T_A m}}$$

$T_A = 4T_B$ ....(i)

and $T_B = (T_A - 1.5)$ eV ....(ii)

from (i) and (ii)

$3T_B$ 1.5 eV $\Rightarrow T_B = 0.5$ eV

$T_B = 0.5$ eV $= 4.5$ eV $-\phi_B$

$\phi = 4$eV

**3.** The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5V and internal resistance of 20Ω, the null point on it is found to be a 1000cm. The resistance of whole wire is :

(1) 120Ω (2) 60Ω

(3) 80Ω (4) 100Ω

**NTA Ans. (4)**

**Sol.** $5 = \lambda\ell$

where $\lambda$ is potential gradient & L is total length of wire.

$$5=\frac{\Delta V}{L}\ell$$

$$\Delta V=\frac{5\times L}{\ell}=5\times\frac{12}{10}=6V = 60\text{ mA}\times R$$

R = 100Ω

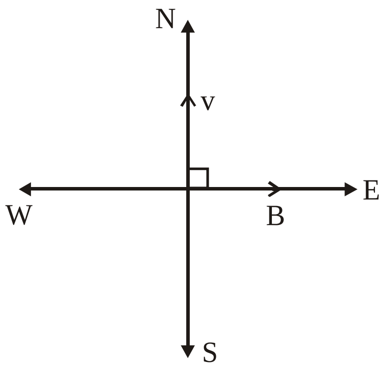
**4.** Photon with kinetic energy of 1MeV moves from south to north. It gets an acceleration of $10^{12}$ m/s² by an applied magnetic field (west to east). The value of magnetic field : (Rest mass of proton is $1.6\times10^{-27}$ kg) :

(1) 71mT (2) 7.1mT

(3) 0.071mT (4) 0.71mT

**NTA Ans. (4)**

**Sol.** $a = \dfrac{qvB}{m}$

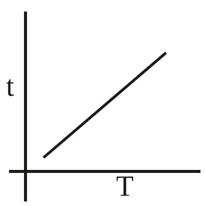


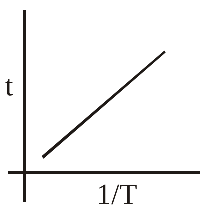

$$B = \frac{ma}{qv} = \frac{ma\sqrt{m}}{\sqrt{2k}}$$

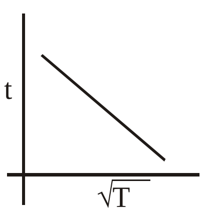
$$= \frac{m^{3/2}a}{e\sqrt{2k}} = \frac{(1.6\times10^{-27})^{3/2}\times10^{12}}{1.6\times10^{-19}\sqrt{2\times1\times10^{6}\times1.6\times10^{-19}}}$$

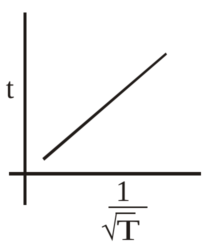
= 0.71 mT

**5.** The plot that depicts the behavior of the mean free time t (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

(1) 


(2) 


(3) 


(4) 


**NTA Ans. (4)**

**Sol.** Mean free time $= \dfrac{\text{Mean free path}}{\text{Average speed}}$

$$= \frac{\dfrac{1}{\sqrt{2}\pi D^2 n}}{\sqrt{\dfrac{8RT}{\pi M_w}}}$$

$$t \propto \frac{1}{\sqrt{T}}$$

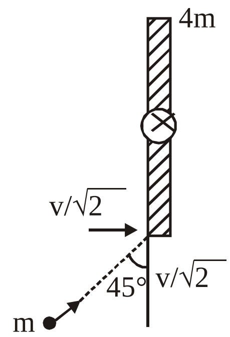
**6.** Consider a uniform rod of mass M = 4m and length $\ell$ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \dfrac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is :

(1) $\dfrac{3}{7\sqrt{2}}\dfrac{v}{\ell}$ (2) $\dfrac{3\sqrt{2}}{7}\dfrac{v}{\ell}$

(3) $\dfrac{4}{7}\dfrac{v}{\ell}$ (4) $\dfrac{3}{7}\dfrac{v}{\ell}$

**NTA Ans. (2)**

**Sol.**




Let angular velocity of the system after collision be $\omega$.

By conservation of angular momentum about the hinge :

$$m\left(\frac{v}{\sqrt{2}}\right)\left(\frac{\ell}{2}\right) = \left[\frac{4m\ell^2}{12} + \frac{m\ell^2}{4}\right]\omega$$

On solving

$$\omega = \frac{3\sqrt{2}}{7}\left(\frac{v}{\ell}\right)$$

**7.** The dimension of stopping potential $V_0$ in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is :

(1) $h^2 G^{3/2} c^{1/3} A^{-1}$ (2) $h^{-2/3} c^{-1/3} G^{4/3} A^{-1}$

(3) $h^{1/3} G^{2/3} c^{1/3} A^{-1}$ (4) $h^{2/3} c^{5/3} G^{1/3} A^{-1}$

**NTA Ans. (4)**

**Sol.** $v_0 = h^x c^y G^z A^w$

$$\frac{ML^2T^{-2}}{AT} = (ML^2T^{-1})^x (LT^{-1})^y (M^{-1}L^3T^{-2})^z A^w$$

$\Rightarrow w = -1$

$(x - z = 1)$

$2x + y + 3x = 2$

$- x - y - 2z = -3$

$2x = 0$

$x = 0$

$z = -1$

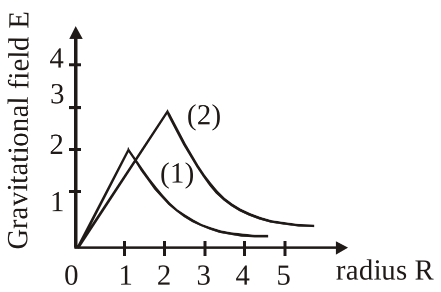
$2 \times 0 + y + 3x - 1 = 2$

$y = 5$

$\Rightarrow v_0 = h^0 c^5 G^{-1} A^{-1}$

So Bonus

**8.** Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses $M_1$ and $M_2$, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is :




(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{6}$

**NTA Ans. (4)**

**Sol.** Gravitational field on the surface of a solid sphere $I_g = \frac{GM}{R^2}$

By the graph

$$\frac{GM_1}{(1)^2} = 2$$

and $\frac{GM_2}{(2)^2} = 3$

On solving

$$\frac{M_1}{M_2} = \frac{1}{6}$$

**9.** In finding the electric field using Gauss Law the formula $|\vec{E}| = \frac{q_{enc}}{\varepsilon_0 |A|}$ is applicable. In the formula $\varepsilon_0$ is permittivity of free space, A is the area of Gaussian surface and $q_{enc}$ is charge enclosed by the Gaussian surface. The equation can be used in which of the following situation?

(1) Only when the Gaussian surface is an equipotential surface.

(2) Only when $|\vec{E}|$ = constant on the surface.

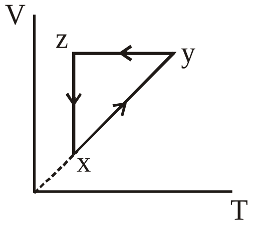
(3) For any choice of Gaussian surface.

(4) Only when the Gaussian surface is an equipotential surface and $|\vec{E}|$ is constant on the surface.

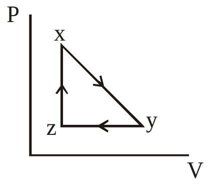
**NTA Ans. (4)**

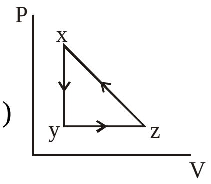
**Sol.** $|\vec{E}|$ should be constant on the surface and the surface should be equipotential.

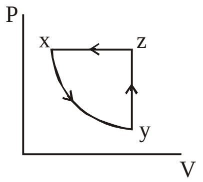
**10.** A thermodynamic cycle xyzx is shown on a V-T diagram.

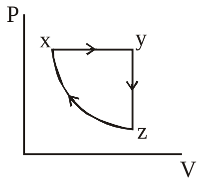



The P-V diagram that best describes this cycle is : (Diagrams are schematic and not to scale)

(1) 


(2) 


(3) 

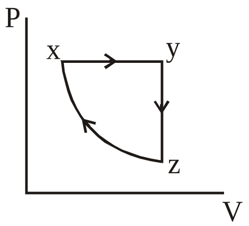

(4) 


**NTA Ans. (4)**

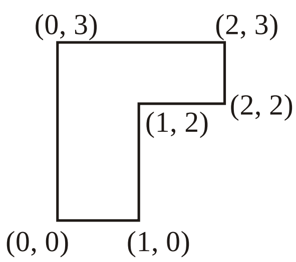
**Sol.** $x \rightarrow y \Rightarrow$ Isobaric

$y \rightarrow z \Rightarrow$ Isochoric

$z \rightarrow x \Rightarrow$ Isothermal




**11.** The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are :



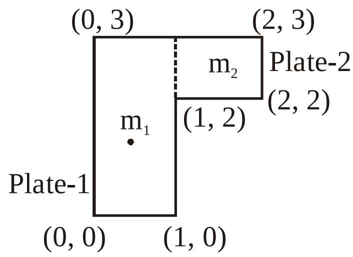

(1) (1.25m, 1.50m) (2) (1m, 1.75m)

(3) (0.75m, 0.75m) (4) (0.75m, 1.75m)

**NTA Ans. (4)**

**Sol.** $m_1 = 3\text{kg}$

$m_2 = 1\text{kg}$




Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3\times0.5 + 1\times1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3\times1.5 + 1\times2.5}{4} = 1.75$$

**12.** The magnifying power of a telescope with tube 60 cm is 5. What is the focal length of its eye piece ?

(1) 30 cm (2) 40 cm

(3) 20 cm (4) 10 cm

**NTA Ans. (4)**

**Sol.** $L = f_0 + f_e = 60$ cm

$$M = \frac{f_0}{f_e} = 5$$

$\Rightarrow f_0 = 5f_e$

$\therefore\ 6f_e = 60$ cm

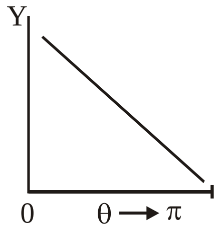
$f_e = 10$ cm

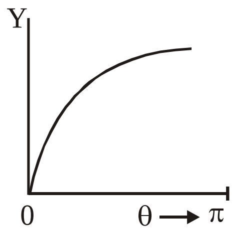
**13.** The graph which depicts the results of Rutherform gold foil experiment with $\alpha$-particales is :

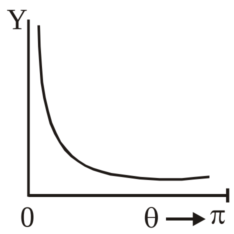
$\theta$ : Scattering angle

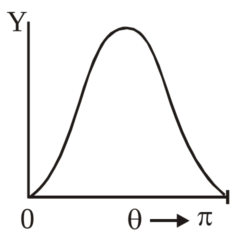
Y : Number of scattered $\alpha$-particles detected

(Plots are schematic and not to scale)

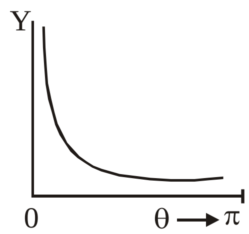
(1) 


(2) 


(3) 


(4) 


**NTA Ans. (3)**

**Sol.** 

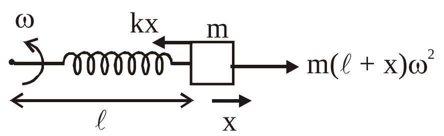

$$Y \propto \frac{1}{\left(\sin\frac{\theta}{2}\right)^4}$$

**14.** A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length $\ell$. The other end is fixed. The system is given an angular speed $\omega$ about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is :

(1) $\frac{m\ell\omega^2}{k+m\omega^2}$ (2) $\frac{m\ell\omega^2}{k-m\omega^2}$

(3) $\frac{m\ell\omega^2}{k-\omega m}$ (4) $\frac{m\ell\omega^2}{k+m\omega}$

**NTA Ans. (2)**

**Sol.** 


$kx = m\ell\omega^2 + mx\omega^2$

$$x = \frac{m\ell\omega^2}{k-m\omega^2}$$

**15.** The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be :

(1) 60° (2) 15° (3) 45° (4) 30°

**NTA Ans. (4)**

**Sol.** $\sin\theta_C = \frac{1}{\mu} = \frac{1}{\sqrt{3\times 4/3}}$

$\theta_C = 30°$

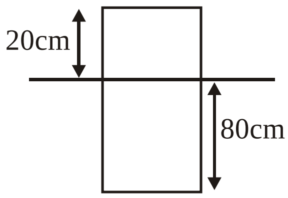
**16.** A leak proof cylinder of length 1m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4°C, relative to the density at T = 0°C is close to :

(1) 1.01 (2) 1.04 (3) 1.03 (4) 1.26

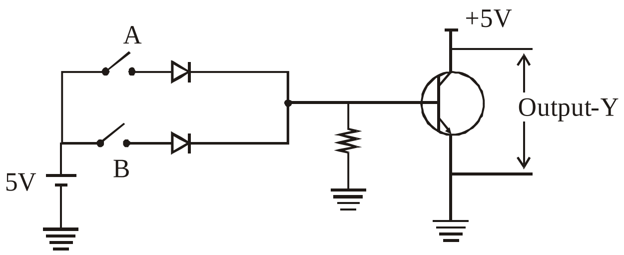
**NTA Ans. (1)**

**Sol.** $m = \rho_0 A\ (80)$ ....(i)

$m = \rho A\ (79)$ ....(ii)




**17.** Boolean relation at the output stage-Y for the following circuit is :



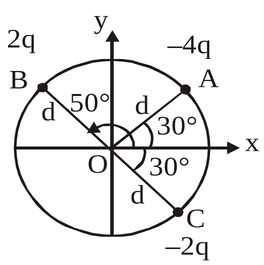

(1) A + B (2) $\bar{A}+\bar{B}$ (3) $\bar{A}\cdot\bar{B}$ (4) $A\cdot B$

**NTA Ans.** **(3)**

**Sol.**

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

**18.** Three charged particle A, B and C with charges –4q, 2q and –2q are present on the circumference of a circle of radius d. the charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is :



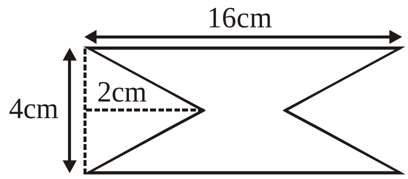

(1) $\frac{2\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$ (2) $\frac{\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$

(3) $\frac{3\sqrt{3}\,q}{4\pi\varepsilon_0 d^2}$ (4) $\frac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$

**NTA Ans.** **(4)**

**Sol.** $E_x = \frac{K(4q)}{R^2}\cos 30° + \frac{K(2q)}{R^2}\cos 30° + \frac{K(2q)}{R^2}\cos 30°$

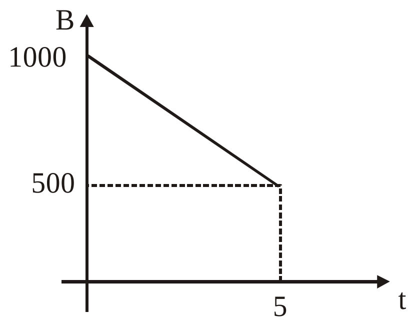
**19.** At time t = 0 magnetic field of 100 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5s, then induced EMF in the loop is :




(1) 36 μV (2) 48 μV (3) 56 μV (4) 28 μV

**NTA Ans.** **(3)**

**Sol.**




$$\frac{dB}{dt} = 100$$

$A = 16 \times 4 - 4 \times 2 = 56\ cm^2$

$$\varepsilon = \frac{dB}{dt}A = 100 \times 10^{-4} \times 56 \times 10^{-4}$$

**20.** Effective capacitance of parallel combination of two capacitors $C_1$ and $C_2$ is 10 μF. When these capacitors are individually connected to a voltage source of 1V, the energy stored in the capacitor $C_2$ is 4 times that of $C_1$. If these capacitors are connected in series, their effective capacitance will be :

(1) 3.2 μF (2) 8.4 μF

(3) 1.6 μF (4) 4.2 μF

**NTA Ans. (3)**

**Sol.** $C_1 + C_2 = 10$ ....(i)

$$\frac{1}{2}C_2V^2 = 4\times\frac{1}{2}C_1V^2$$

$\therefore$ $C_2 = 4C_1$ ....(ii)

$\therefore$ $C_1 = 2$ & $C_2 = 8$

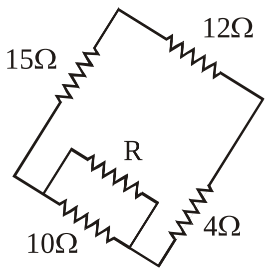
For series combination

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = 1.6$$

**21.** Four resistances of 15Ω, 12Ω, 4Ω and 10Ω respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10Ω to balance the network is ________ Ω.

**NTA Ans. (10.00)**

**Sol.**




Let the resistance to be connected is R.

For balanced wheatstone bridge,

$$15 \times 4 = 12 \times \frac{10R}{10+R}$$

$\Rightarrow R = 10\Omega$

**22.** A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ——.

**NTA Ans. (60.00)**

**Sol.** Using Lens-Maker's formula :

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f} = (1.5-1)\left(\frac{1}{30}-0\right)$$

f = 60 cm

**23.** A body A, of mass m = 0.1 kg has an initial velocity of $3\hat{i}\,ms^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}\,ms^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i}+\hat{j})$. The energy of B after collision is written as $\frac{x}{10}J$.

The value of x is __________.

**NTA Ans. (16.00)**

**Sol.** By conservation of linear momentum :

$$(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)(4)(\hat{i}+\hat{j}) + (0.1)\vec{v}$$

$$\Rightarrow \vec{v} = -\hat{i} + \hat{j}$$

$\therefore$ Speed of B after collision $|\vec{v}| = \sqrt{2}$

Now, kinetic energy = $\frac{1}{2}mV^2 = \frac{1}{2}(0.1)(2) = \frac{1}{10}$

$\therefore$ x = 1

**24.** A particle is moving along the x-axis with its coordinate with the time 't' given be $x(t) = 10 + 8t - 3t^2$. Another particle is moving the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At t = 1s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is ___________.

**NTA Ans. (13.00)**

**Sol.** $x = 10 + 8t - 3t^2$

$v_x = 8 - 6t$

$(v_x)_{t=1} = 2\hat{i}$

$y = 5 - 8t^3$

$v_y = -24t^2$

$(v_y)_{t=1} = -24\hat{j}$

Now

$$\sqrt{v} = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$$

$\therefore$ v = 580 $m^2/s^2$

**25.** A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is _________ Hz.

**NTA Ans. (106.00)**

**Sol.** $v_s = \sqrt{\frac{\gamma P}{\rho}}$

$$\frac{v_{gas}}{v_{air}} = \sqrt{\frac{\rho_{air}}{\rho_{gas}}} \qquad \Rightarrow \frac{v_{gas}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow v_{gas} = \frac{300}{\sqrt{2}}$$

$$\therefore v_{gas} = 150\sqrt{2}$$

Now $n_2 - n_1 = \frac{v_{gas}}{2\ell} = \frac{150\sqrt{2}}{2(1)} = 75\sqrt{2}$

$\Rightarrow$ $\Delta n$ = 106.06 Hz

# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM**

## CHEMISTRY — TEST PAPER WITH SOLUTION

**1.** A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63°C while the other boils at 60°C. What is the best way to seprate the two liquids and which one will be distilled out first?

(1) simple distillation, 3-methylpentane

(2) simple distillation, isohexane

(3) fractional distillation, isohexane

(4) fractional distillation, 3-methylpentane

**NTA Ans. (3)**

**Sol.** Liquid which have less difference in boiling point can be isolated by fractional distillation and liquid with less boiling point will be isolated first.

**2.** The first ionization energy (in kJ/mol) of Na, Mg, Al and Si respectively, are :

(1) 496, 737, 577, 786

(2) 786, 737, 577, 496

(3) 496, 577, 737, 786

(4) 496, 577, 786, 737

**NTA Ans. (1)**

**Sol.** Elecronic configuration of Na = [Ne] $3s^1$

Mg = [Ne] $3s^2$

Al = [Ne] $3s^2 3p^1$

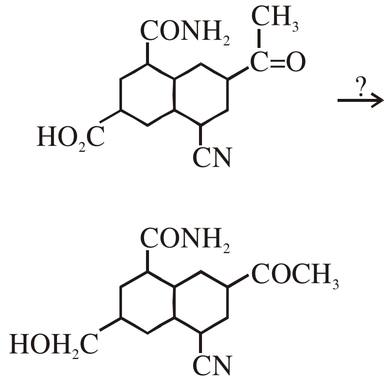
Si = [Ne] $3s^2 3p^2$

So order of first ionisation energy is

$$\underset{496}{Na} < \underset{737}{Mg} > \underset{577}{Al} < \underset{786}{Si} \text{ kj/mol}$$

Na < Al < Mg < Si ($IE_1$ order)

**3.** The most suitable reagent for the given conversion is :

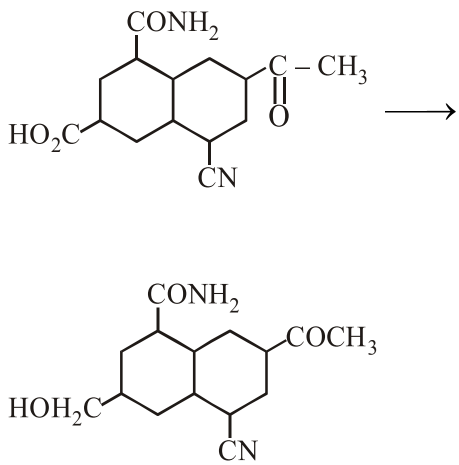


(1) $LiAlH_4$ (2) $NaBH_4$

(3) $H_2/Pd$ (4) $B_2H_6$

**NTA Ans. (4)**

**Sol.**



Most suitable reagent for given conversion is $B_2H_6$ (electrophilic reducing agent)

**4.** The third ionization enthalpy is minimum for :

(1) Fe (2) Ni (3) Co (4) Mn

**NTA Ans. (1)**

**Sol.** Electronic configuration of

| | $_{25}Mn$ | $_{26}Fe$ | $_{27}Co$ | $_{28}Ni$ |
|---|---|---|---|---|
| $M =$ | $[Ar]3d^54s^2$ | $[Ar]3d^64s^2$ | $[Ar]3d^74s^2$ | $[Ar]3d^84s^2$ |
| $M^{2+} =$ | $[Ar]3d^54s^0$ | $[Ar]3d^64s^0$ | $[Ar]3d^74s^0$ | $[Ar]3d^84s^0$ |

So third ionisation energy is minimum for Fe.

**5.** The predominant intermolecular forces present in ethyl acetate, a liquid, are :

(1) hydrogen bonding and London dispersion

(2) Dipole-dipole and hydrogen bonding

(3) London dispersion and dipole-dipole

(4) London dispersion, dipole-dipole and hydrogen bonding

**NTA Ans. (3)**

**Sol.** Ethyl acetate $\left(H_3C-\underset{\underset{O}{\|}}{C}-O-CH_2-CH_3\right)$ is polar molecule. Hence there will be dipole-dipole attraction and london dispersion forces are present.

**6.** The strength of an aqueous NaOH solution is most accurately determined by titrating :

(Note : consider that an appropriate indicator is used)

(1) Aq. NaOH in a volumetric flask and concentrated $H_2SO_4$ in a conical flask

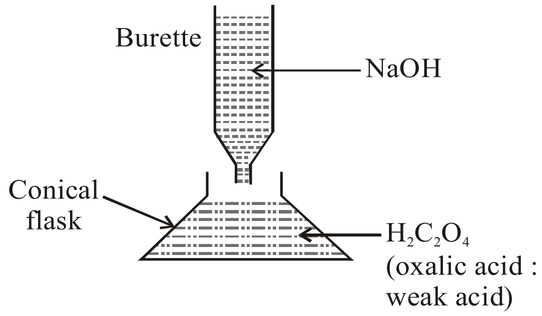
(2) Aq. NaOH in a pipette and aqueous oxalic acid in a burette

(3) Aq. NaOH in a burette and concentrated $H_2SO_4$ in a conical flask

(4) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask

**NTA Ans. (4)**

**Sol.**




**7.** The complex that can show fac-and mer-isomers is :

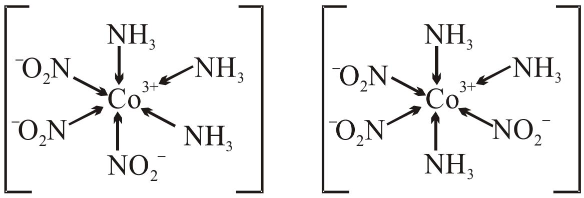
(1) $[Pt(NH_3)_2Cl_2]$ (2) $[Co(NH_3)_4Cl_2]^+$

(3) $[Co(NH_3)_3(NO_2)_3]$ (4) $[CoCl_2(en)_2]$

**NTA Ans. (3)**

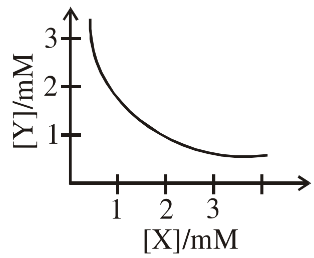
**Sol.** $[Ma_3b_3]$ type complex shows fac and mer isomerism.

$[Co(NH_3)_3(NO_2)_3]$




fac-isomer mer-isomer

**8.** The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively :




(1) $X_2Y$, $2\times10^{-9}M^3$

(2) $XY_2$, $1\times10^{-9}M^3$

(3) $XY_2$, $4\times10^{-9}M^3$

(4) $XY$, $2\times10^{-6}M^3$

**NTA Ans. (3)**

**Sol.** From the graph & dimensions salt is : $XY_2$

$[X] = 1 \times 10^{-3}M$

$[Y] = 2 \times 10^{-3}M$

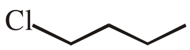
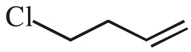
$$XY_2(s) \rightleftharpoons X^{2+}_{(aq.)} + 2Y^{-}_{(aq.)}$$

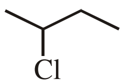
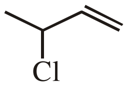
$ksp = [X^{2+}]\,[Y^-]^2$

$= (10^{-3})\,(2 \times 10^{-3})^2$

$= 4 \times 10^{-9}\ M^3$

**9.** The decreasing order of reactivity towards dehydrohalogenation ($E_1$) reaction of the following compounds is :

(A)  (B) 

(C)  (D) 

(1) B > D > A > C

(2) B > D > C > A

(3) D > B > C > A

(4) B > A > D > C

**NTA Ans. (3)**

**Sol.** Reactivity D > B > C > A

Carbocation formed from D is most stable

Carbocation formed from A is least stable

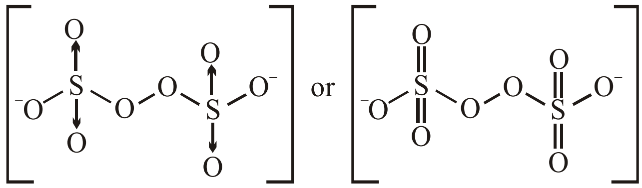
**10.** The number of bonds between sulphur and oxygen atoms in $S_2O_8^{2-}$ and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are :

(1) 4 and 8 (2) 4 and 6

(3) 8 and 8 (4) 8 and 6

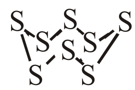
**NTA Ans. (3)**

**Sol.** $S_2O_8^{2-}$ :



8 bonds are present between sulphur and oxygen. (It is best answer in given options)

**Rhombic sulphur :**

**($S_8$)** 

8 bonds are present between sulphur and sulphur atoms.

**11.** The rate of a certain biochemical reaction at physiological temperature (T) occurs $10^6$ times faster with enzyme than without. The change in the activation energy upon adding enzyme is :

(1) –6RT (2) +6RT

(3) +6(2.303)RT (4) –6(2.303)RT

**NTA Ans. (4)**

**Sol.** $K = Ae^{\frac{-E_a}{RT}}$

$$K' = Ae^{\frac{-E_a'}{RT}} = 10^6 K$$

$$Ae^{\frac{-E'}{RT}} = 10^6 \times Ae^{\frac{-E_a}{RT}}$$

$$\frac{-E_a'}{RT} = \frac{-E_a}{RT} + \ln 10^6$$

$$E_a' = E_a - RT\ \ln 10^6$$

$$E_a' - E_a = -RT\ \ln 10^6$$

$= -\ 6RT \times 2.303$

**12.** Which of the following statement is not true for glucose?

(1) The pentaacetate of glucose does not react with hydroxylamine to give oxime

(2) Glucose gives Schiff's test for aldehyde

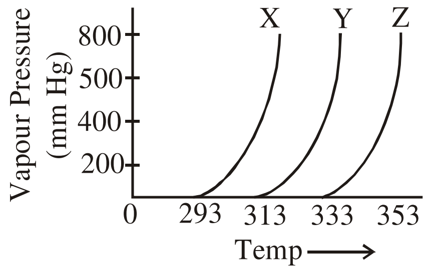
(3) Glucose exists in two crystalline forms $\alpha$ and $\beta$

(4) Glucose reacts with hydroxylamine to form oxime

**NTA Ans. (2)**

**Sol.** Glucose gives negative test with Schiff reagent

**13.** A graph of vapour pressure and temperature for three different liquids X, Y and Z is shown below :




The following inferences are made :

(A) X has higher intermolecular interactions compared to Y.

(B) X has lower intermolecular interactions compared to Y.

(C) Z has lower intermolecular interactions compared to Y.

The correct inference(s) is/are :

(1) A (2) (C)

(3) (B) (4) (A) and (C)

**NTA Ans. (3)**

**Sol.** Order of B.P. is : Z > Y > X

Order of vapour pressure : Z < Y < X

order of intermolecular interaction : Z > Y > X.

**14.** Among the gases (a) - (e), the gases that cause greenhouse effect are :

(a) $CO_2$ (b) $H_2O$

(c) CFCs (d) $O_2$

(e) $O_3$

(1) (a), (b), (c) and (d)

(2) (a), (c), (d) and (e)

(3) (a) and (d)

(4) (a), (b), (c) and (e)

**NTA Ans. (4)**

**Sol.** $CO_2$, $H_2O$, CFCs and $O_3$ are green house gases.

**15.** As per Hardy-Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order :

(1) $AlCl_3 > K_3[Fe(CN)_6] > K_2CrO_4 > KBr = KNO_3$

(2) $K_3[Fe(CN)_6] < K_2CrO_4 < AlCl_3 < KBr < KNO_3$

(3) $K_3[Fe(CN)_6] > AlCl_3 > K_2CrO_4 > KBr > KNO_3$

(4) $K_3[Fe(CN)_6] < K_2CrO_4 < KBr = KNO_3 = AlCl_3$

**NTA Ans. (4)**

**Sol.** Since, $Fe(OH)_3$ is positively charged sol, hence, anionic charge will flocculate

As per Hardy Schulze rules coagulation power of anion follows the order :

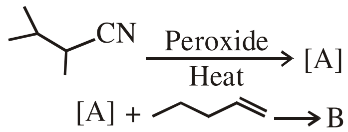
$Fe(CN)_6^{3-} > CrO_4^{2-} > Cl^- = Br^- = NO_3^-$

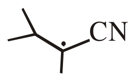
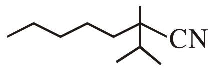
Higher the coagulation power lower will be its flocculation value

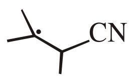
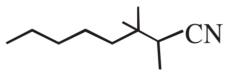
therefore order will be :

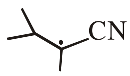
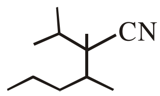
$Fe(CN)_6^{3-} < CrO_4^{2-} < Cl^- = Br^- = NO_3^-$

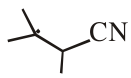
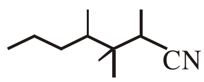
**16.** The major products A and B in the following reactions are :




(1) A =  and B = 

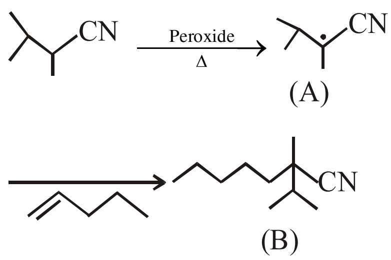
(2) A =  and B = 

(3) A =  and B = 

(4) A =  and B = 

**NTA Ans. (1)**

**Sol.**




(A)

(B)

**17.** For the Balmer series in the spectrum of H atom, $\bar{\nu} = R_H \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\}$, the correct statements among (I) and (IV) are :

(I) As wavelength decreases, the lines in the series converge

(II) The integer $n_1$ is equal to 2

(III) The lines of longest wavelength corresponds to $n_2 = 3$

(IV) The ionization energy of hydrogen can be calculated from wave number of these lines

(1) (II), (III), (IV) (2) (I), (II), (III)

(3) (I), (III), (IV) (4) (I), (II), (IV)

**NTA Ans. (2)**

**Sol.** For balmer : $n_1 = 2$, $n_2 = 3, 4, 5, \ldots \infty$

$$\bar{\nu} = \frac{1}{\lambda} = R_H \left[ \frac{1}{2^2} - \frac{1}{n_2^2} \right]$$

$$\frac{1}{\lambda_{\text{longest}}} = R_H \left[ \frac{1}{2^2} - \frac{1}{3^2} \right]$$

Ans.(2)

**18.** Arrange the following compounds in increasing order of C–OH bond length :

methanol, phenol, p-ethoxyphenol

(1) phenol < methanol < p-ethoxyphenol

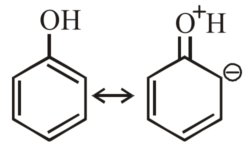
(2) phenol < p-ethoxyphenol < methanol

(3) methanol < p-ethoxyphenol < phenol

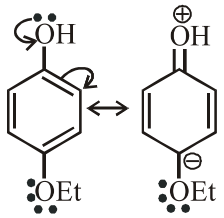
(4) methanol < phenol < p-ethoxyphenol

**NTA Ans. (2)**

**Sol.** $H_3C - OH$ (100% single bond)

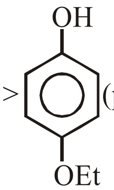
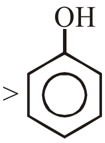


C–OH bond has partial double bond character

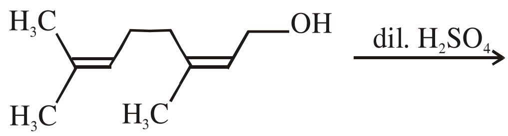


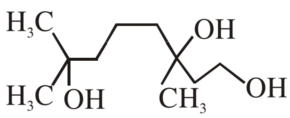
(+M)

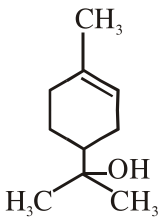
(C–OH bond has some double bond character but double bond character is less)

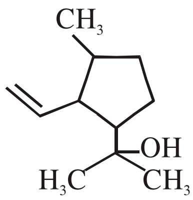
**Ans.** $CH_3OH$ >  (p-ethoxyphenol) > 

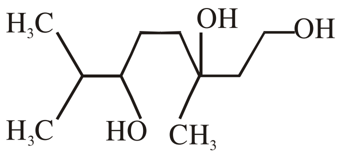
**19.** The major product of the following reaction is :



(1) 

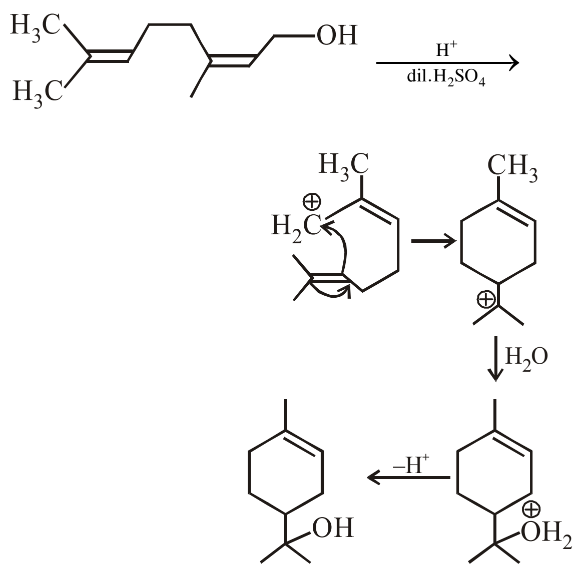
(2) 

(3) 

(4) 

**NTA Ans. (2)**

**Sol.**




**Terpineol**

**20.** When gypsum is heated to 393 K, it forms :

(1) Dead burnt plaster

(2) Anhydrous $CaSO_4$

(3) $CaSO_4.5H_2O$

(4) $CaSO_4.0.5H_2O$

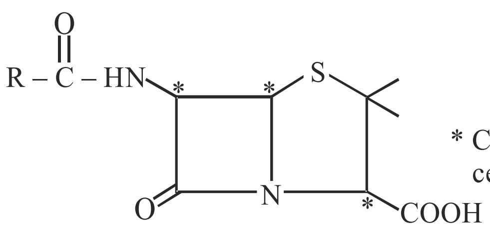
**NTA Ans. (4)**

**Sol.** $\underset{\text{Gypsum}}{CaSO_4.2H_2O} \xrightarrow{393\text{K}} \underset{\text{Plaster of paris}}{CaSO_4.\frac{1}{2}H_2O} + \frac{3}{2}H_2O$

**21.** The number of chiral centres in penicillin is ______.

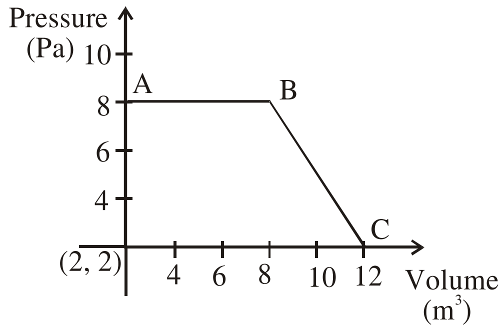
**NTA Ans. (3.00 to 3.00)**

**Sol.** The structure of penicillin is




* Chiral center = 3

**22.** The magnitude of work done by a gas that undergoes a reversible expansion along the path ABC shown in the figure is ______




**NTA Ans. (48.00 to 48.00)**

**Sol.** **22.** Area enclosed under

P V curve = 48

= 48 Joule

**23.** The volume (in mL) of 0.125 M $AgNO_3$ required to quantitatively precipitate chloride ions in 0.3 g of $[Co(NH_3)_6]Cl_3$ is ______.

$M_{[Co(NH_3)_6]Cl_3}$ = 267.46 g/mol

$M_{AgNO_3}$ = 169.87 g/mol

**NTA Ans. (26.80 to 27.00)**

**Sol.** Number of moles of $Cl^-$ precipitated in $[Co(NH_3)_6]Cl_3$ is equal to number of moles of $AgNO_3$ used.

$$\frac{0.3}{267.46} \times 3 = \frac{0.125 \times V}{1000}$$

where V is volume of $AgNO_3$ (in mL)

V = 26.92 mL

**24.** What would be the electrode potential for the given half cell reaction at pH = 5 ?______

$2H_2O \rightarrow O_2 + 4H^{\oplus} + 4e^-$ ; $E^0_{red}$ = 1.23 V

(R = 8.314 J mol$^{-1}$ K$^{-1}$ ; Temp = 298 K; oxygen under std. atm. pressure of 1 bar)

**NTA Ans. (1.52 to 1.53)**

**Sol.** $O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O(l)$ ; $E^0_{red.} = 1.23V$

From nernst equation

$$E_{cell} = E^0_{cell} - \frac{RT}{nF}\ln Q$$

at 1 bar & 298 K

$$\frac{2.303RT}{F} = 0.059$$

pH = 5 $\Rightarrow [H^+] = 10^{-5}$ M

$$E_{cell} = 1.23 - \frac{0.059}{4}\log[H^+]^4$$

$$E_{cell} = 1.23 - \frac{0.059}{4}\log(10^{-5})^4$$

= 1.23 + 0.295 = 1.525 V

**25.** Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is _____.
Atomic weight : Fe = 55.85 ; S = 32.0 ; O = 16.00

**NTA Ans. (4.95 to 4.97)**

**Sol.** $FeSO_4.7H_2O$ (M = 277.85)

$$\text{ppm} = \frac{\text{wt. of Fe}}{\text{wt. of wheat}} \times 10^6$$

let the wt. of salt be = w gm

$$\text{moles} = \frac{w}{277.85}$$

$$\text{wt. of Fe} = \left(\frac{W}{277.85} \times 55.85\right)\text{gm}$$

$$10 = \frac{\frac{W}{277.85} \times 55.85}{10^5} \times 10^6$$

$$W = \frac{277.85}{55.85} = 4.97$$

# FINAL JEE–MAIN EXAMINATION – JANUARY, 2020

**(Held On Wednesday 08th JANUARY, 2020) TIME : 9 : 30 AM to 12 : 30 PM**

## MATHEMATICS

## TEST PAPER WITH ANSWER & SOLUTION

**1.** Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a ponit P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0,\beta)$, then $\beta$ is equal to

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{\sqrt{2}}{3}$

**NTA Ans. (4)**

**Sol.** Any normal to the ellipse is

$$\frac{x\sec\theta}{\sqrt{2}} - y\operatorname{cosec}\theta = -\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(\frac{-\cos\theta}{\sqrt{2}}\right)} + \frac{y}{\left(\frac{\sin\theta}{2}\right)} = 1$$

$$\Rightarrow \frac{\cos\theta}{\sqrt{2}} = \frac{1}{3\sqrt{2}} \text{ and } \frac{\sin\theta}{2} = \beta$$

$$\Rightarrow \beta = \frac{\sqrt{2}}{3}$$

**2.** Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is

(1) 0 (2) 3 (3) 2 (4) 4

**NTA Ans. (2)**

**Sol.** $f(x) = \frac{2(2^x + 2^{-x}) + (3^x + 3^{-x})}{2} \geq 3$

(A.M $\geq$ G.M)

**3.** Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}, \vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If $\theta$ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be

(1) $\frac{7}{6\sqrt{3}}$ (2) $\frac{5}{7}$ (3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{5}{3\sqrt{3}}$

**NTA Ans. (1)**

**Sol.** $$\left\|\begin{matrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{matrix}\right\| = 1 \Rightarrow \lambda = 2, 4$$

Now, $\cos\theta = \frac{\vec{u}.\vec{w}}{|\vec{u}||\vec{w}|}$

$$= \frac{5}{\sqrt{6}\sqrt{6}} \text{ or } \frac{7}{\sqrt{6}\sqrt{18}} = \frac{5}{6} \text{ or } \frac{7}{6\sqrt{3}}$$

**4.** If a,b and c are the greatest value of ${}^{19}C_p, {}^{20}C_q$ and ${}^{21}C_r$ respectively, then

(1) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (2) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

(3) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$ (4) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

**NTA Ans. (4)**

**Sol.** $a = {}^{19}C_{10}$, $b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$\Rightarrow a = {}^{19}C_9$, $b = 2({}^{19}C_9)$ and $c = \frac{21}{11}\left({}^{20}C_{10}\right)$

$\Rightarrow b = 2a$ and $c = \frac{21}{11}b = \frac{42a}{11}$

$\Rightarrow a : b : c = a : 2a : \frac{42a}{11} = 11 : 22 : 42$

**5.** Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$, $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}(f(x))\right)$ and $y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to

(1) $\frac{5\pi}{6}$ (2) $-\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

**NTA Ans. (1)**

**Sol.** Let $\tan^{-1}x = \theta$, $\theta \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$$f(x) = (\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta = \frac{2x}{1+x^2}$$

Now, $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= -\frac{1}{1+x^2}, |x| > 1$$

Since, we can integrate only in the continuous interval. So we have to take integral in two cases separtely namely for x < –1 and for x > 1.

$$\Rightarrow y = \begin{cases} -\tan^{-1}x + c_1 & ; \quad x > 1 \\ -\tan^{-1}x + c_2 & ; \quad x < -1 \end{cases}$$

so, $c_1 = \frac{\pi}{2}$ as $y(\sqrt{3}) = \frac{\pi}{6}$

But we cannot find $c_2$ as we do not have any other additional information for x < –1. So, all of the given options may be correct as $c_2$ is unknown so, it should be bonus.

**6.** $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to

(1) $\frac{1}{e}$ (2) $e^2$

(3) e (4) $\frac{1}{e^2}$

**NTA Ans. (4)**

**Sol.** Required limit $= e^{\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}}$

$$= e^{\lim\limits_{x\to 0}\left(\frac{-4}{7x^2+2}\right)} = \frac{1}{e^2}$$

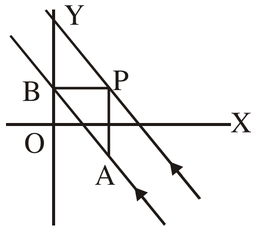
**7.** Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of $\Delta PAB$ = 5 sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of $\lambda$ is

(1) 1 (2) 4

(3) 3 (4) –3

**NTA Ans. (3)**

**Sol.** $\overleftrightarrow{AB}$ : 3x + y – 2 = 0




Also, $\frac{1}{2} \times \sqrt{10} \times h = 5$

$\Rightarrow h = \sqrt{10}$

$$\Rightarrow \frac{|4\lambda - 2|}{\sqrt{10}} = \sqrt{10} \Rightarrow \lambda = 3, -2$$

**8.** The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 resepectively. Each of these 10 observations is multiplied by p and then reduced by q, where p ≠ 0 and q ≠ 0. If the new mean and new s.d. become half of their original values, then q is equal to

(1) –20 (2) 10 (3) –10 (4) –5

**NTA Ans. (1)**

**Sol.** 20p – q = 10 ...(i)

and 2|p| = 1 $\Rightarrow p = \pm\frac{1}{2}$ ...(ii)

so, $p = -\frac{1}{2}$ and q = –20

**9.** Let y = y(x) be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$.

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to

(1) $-\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

**NTA Ans. (2)**

**Sol.** $\dfrac{dy}{dx}=-\dfrac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ so, $\dfrac{dy}{\sqrt{1-y^2}}+\dfrac{dx}{\sqrt{1-x^2}}=0$

Integrating, $\sin^{-1}x + \sin^{-1}y = c$

so, $\dfrac{\pi}{6}+\dfrac{\pi}{3}=c$

Hence, $\sin^{-1}x + \sin^{-1}y = \dfrac{\pi}{2}$

Put $x=-\dfrac{1}{\sqrt{2}}, \sin^{-1}y=\dfrac{3\pi}{4}$ (Not possible)

**10.** If the equation, $x^2 + bx + 45 = 0$ $(b \in R)$ has conjugate complex roots and they satisfy $|z+1|=2\sqrt{10}$, then

(1) $b^2 - b = 42$

(2) $b^2 + b = 12$

(3) $b^2 + b = 72$

(4) $b^2 - b = 30$

**NTA Ans. (4)**

**Sol.** Assuming z is a root of the given equation,

$$z=\frac{-b\pm i\sqrt{180-b^2}}{2}$$

so, $\left(1-\dfrac{b}{2}\right)^2+\dfrac{180-b^2}{4}=40$

$\Rightarrow -4b + 184 = 160 \Rightarrow b = 6$

**11.** For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, $C_1$ and $C_2$, and the area of $\Delta OQR=\dfrac{1}{2}$, then 'a' satisfies the equation

(1) $x^6 - 12x^3 + 4 = 0$

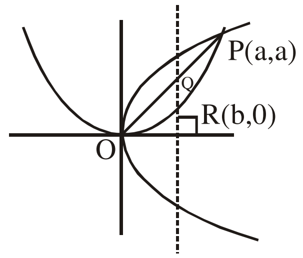
(2) $x^6 - 12x^3 - 4 = 0$

(3) $x^6 + 6x^3 - 4 = 0$

(4) $x^6 - 6x^3 + 4 = 0$

**NTA Ans. (1)**

**Sol.**




$$\int_0^b\left(\sqrt{ax}-\frac{x^2}{a}\right)dx=\frac{1}{2}\times\frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3}$$

$$\Rightarrow\left[\frac{2\sqrt{a}}{3}x^{3/2}-\frac{x^3}{3a}\right]_0^b=\frac{a^2}{6}$$

$$\Rightarrow\frac{2\sqrt{a}}{3}b^{3/2}-\frac{b^3}{3a}=\frac{a^2}{6} \qquad \text{...(i)}$$

Also, $\dfrac{1}{2}\times b^2=\dfrac{1}{2}\Rightarrow b=1$

so, $\dfrac{2\sqrt{a}}{3}-\dfrac{1}{3a}=\dfrac{a^2}{6}\Rightarrow a^3-4a^{3/2}+2=0$

$\Rightarrow a^6 + 4a^3 + 4 = 16a^3 \Rightarrow a^6 - 12a^3 + 4 = 0$

**12.** Which one of the following is a tautology ?

(1) $P\wedge(P\vee Q)$

(2) $P\vee(P\wedge Q)$

(3) $Q\rightarrow(P\wedge(P\rightarrow Q))$

(4) $(P\wedge(P\rightarrow Q))\rightarrow Q$

**NTA Ans. (4)**

**Sol.** (1) $P\wedge(P\vee Q)\equiv P$

(2) $P\vee(P\wedge Q)\equiv P$

(3) $Q\rightarrow(P\wedge(P\rightarrow Q))$

$\equiv Q\rightarrow(P\wedge(\sim P\vee Q))\equiv Q\rightarrow(P\wedge Q)$

$\equiv(\sim Q)\vee(P\wedge Q)\equiv(P\vee(\sim Q))$

(4) $(P\wedge(P\rightarrow Q))\rightarrow Q$

$\equiv(P\wedge(\sim P\vee Q))\rightarrow Q\equiv(P\wedge Q)\rightarrow Q$

$\equiv((\sim P)\vee(\sim Q))\vee Q\equiv(\sim P)\vee t\equiv t$

**13.** The locus of a point which divides the line segment joining the point (0,–1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is-

(1) $9x^2 - 3y = 2$ (2) $9x^2 - 12y = 8$

(3) $x^2 - 3y = 2$ (4) $4x^2 - 3y = 2$

**NTA Ans. (2)**

**Sol.** 1 2

A(0,–1) P(h,k) Q(2t,t²)

$\Rightarrow 3h = 2t$ and $3k = t^2 - 2$

$$\Rightarrow 3y = \left(\frac{3x}{2}\right)^2 - 2 \Rightarrow 12y = 9x^2 - 8$$

**14.** If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3,4], where $\alpha \in R$, then $f''(c)$ is equal to

(1) $\frac{\sqrt{3}}{7}$ (2) $\frac{1}{12}$ (3) $-\frac{1}{24}$ (4) $-\frac{1}{12}$

**NTA Ans. (2)**

**Sol.** $\frac{9+\alpha}{21} = \frac{16+\alpha}{28} \Rightarrow \alpha = 12$

Also, $f'(x) = \frac{7x}{x^2+12} \times \frac{x^2-12}{7x^2} = \frac{x^2-12}{x(x^2+12)}$

Hence, $c = 2\sqrt{3}$

Now, $f''(c) = \frac{1}{12}$

**15.** For which of the following ordered pairs $(\mu,\delta)$, the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent ?

(1) (1,0) (2) (4,6)

(3) (3,4) (4) (4,3)

**NTA Ans. (4)**

**Sol.** $2 \times (ii) - 2 \times (i) - (iii)$ : -

$0 = 2\mu - 2 - \delta$

$\Rightarrow \delta = 2(\mu - 1)$

**16.** Let A and B be two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ?

(1) $P(A/B) = \frac{2}{3}$

(2) $P(A/(A \cup B)) = \frac{1}{4}$

(3) $P(A/B') = \frac{1}{3}$

(4) $P(A'/B') = \frac{1}{3}$

**NTA Ans. (3)**

**Sol.** (1) $P(A/B) = P(A) = \frac{1}{3}$

(2) $P(A/(A \cup B)) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$

$$= \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6} - \frac{1}{18}} = \frac{3}{4}$$

(3) $P(A/B') = P(A) = \frac{1}{3}$

(4) $P(A'/B') = P(A') = \frac{2}{3}$

**17.** The inverse function of

$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1,1)$, is

(1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

**NTA Ans. (3)**

**Sol.** $f(x)=y=\frac{8^{4x}-1}{8^{4x}+1}=1-\frac{2}{8^{4x}+1}$

so, $8^{4x}+1=\frac{2}{1-y}\Rightarrow 8^{4x}=\frac{1+y}{1-y}$

$\Rightarrow x=\ell n\left(\frac{1+y}{1-y}\right)\times\frac{1}{4\ell n8}=f^{-1}(y)$

Hence, $f^{-1}(x)=\frac{1}{4}\log_8 e\ell n\left(\frac{1+x}{1-x}\right)$

**18.** If $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}=f(x)\left(1+\sin^6 x\right)^{1/\lambda}+c$

where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) –2 (2) $-\frac{9}{8}$ (3) 2 (4) $\frac{9}{8}$

**NTA Ans. (1)**

**Sol.** $\int\frac{\cos x\,dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}=\frac{-6}{-6}\int\frac{\cos x\,dx}{\sin^7 x\left(\frac{1}{\sin^6 x}+1\right)^{2/3}}$

$=-\frac{1}{6}\times 3\left(\frac{1}{\sin^6 x}+1\right)^{\frac{1}{3}}+c$

$=-\frac{1}{2}\frac{\left(1+\sin^6 x\right)^{\frac{1}{3}}}{\sin^2 x}+c$

Hence, $\lambda$ = 3 and $f(x)=-\frac{1}{2\sin^2 x}$

so, $\lambda f\left(\frac{\pi}{3}\right)=-2$

**REMARK :** Technically, this question should be marked as bonus. Because $f(x)$ and $\lambda$ cannot be found uniquely.

For example, another such $f(x)$ and $\lambda$ can be

$-\frac{\left(1+\sin^6 x\right)^{\frac{1}{6}}}{2\sin^2 x}$ and 6 respectively.

**19.** The shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and

$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$ (3) 3 (4) $2\sqrt{30}$

**NTA Ans. (2)**

**Sol.** Shortest distance $=\frac{\left\|\begin{matrix}6 & 15 & -3\\ 3 & -1 & 1\\ -3 & 2 & 4\end{matrix}\right\|}{\sqrt{11\times 29-49}}=\frac{270}{\sqrt{270}}$

$=\sqrt{270}=3\sqrt{30}$

**20.** Let $f(x)$ = $x\cos^{-1}(-\sin|x|)$, $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, then which of the following is true ?

(1) $f'$ is decreasing in $\left(-\frac{\pi}{2},0\right)$ and increasing in $\left(0,\frac{\pi}{2}\right)$

(2) $f$ is not differentiable at x = 0

(3) $f'(0)=-\frac{\pi}{2}$

(4) $f'$ is increasing in $\left(-\frac{\pi}{2},0\right)$ and decreasing in $\left(0,\frac{\pi}{2}\right)$

**NTA Ans. (1)**

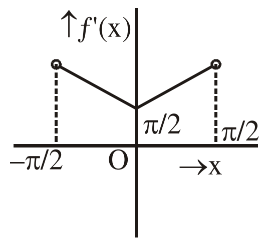
**Sol.** $f(x)$ is an odd function.

Now, if $x \geq 0$, then $f(x) = x\cos^{-1}(-\sin x)$

$$= x\left(\frac{\pi}{2} - \sin^{-1}(-\sin x)\right) = x\left(\frac{\pi}{2} + x\right)$$

Hence, $$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & ; \quad x \in \left[0, \frac{\pi}{2}\right] \\ x\left(\frac{\pi}{2} - x\right) & ; \quad x \in \left[-\frac{\pi}{2}, 0\right) \end{cases}$$

so, $$f'(x) = \begin{cases} \frac{\pi}{2} + 2x & ; \quad x \in \left[0, \frac{\pi}{2}\right) \\ \frac{\pi}{2} - 2x & ; \quad x \in \left(-\frac{\pi}{2}, 0\right) \end{cases}$$




**21.** The number of all 3 × 3 matrices A, with enteries from the set {–1,0,1} such that the sum of the diagonal elements of $AA^T$ is 3, is

**NTA Ans. (672.00)**

**Sol.** trace $(AA^T) = \Sigma a_{ij}^2 = 3$

Hence, number of such matrices

$= {}^9C_3 \times 2^3 = 672.00$

**22.** The least positive value of 'a' for which the equation $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

**NTA Ans. (8.00)**

**Sol.** $D \geq 0 \Rightarrow (a - 10)^2 - 4 \times 2 \times \left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow a^2 - 4a - 32 \geq 0$

$\Rightarrow a \in (-\infty, 4] \cup [8,\infty)$

**23.** Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to

**NTA Ans. (4.00)**

**Sol.** Let $P(\alpha,\beta)$

so, $\beta^2 - 3\alpha^2 + \beta + 10 = 0$ ...(i)

Now, $2yy' - 6x + y' = 0$

$\Rightarrow m = \frac{6\alpha}{2\beta + 1}$ ....(ii)

Also, $\frac{\beta - \frac{3}{2}}{\alpha} = -\frac{1}{m}$

$\Rightarrow \frac{2\beta - 3}{2\alpha} = -\frac{(2\beta + 1)}{6\alpha}$ (from (ii))

$\Rightarrow \beta = 1 \Rightarrow \alpha^2 = 4$ (from (1))

Hence, $|m| = \frac{12}{3} = 4.00$

**24.** The sum $\sum_{k=1}^{20}(1+2+3+...+k)$ is ....

**NTA Ans. (1540.00)**

**Sol.** $\sum_{k=1}^{20}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{20}\frac{k(k+1)(k+2)-(k-1)k(k+1)}{3}$

$=\frac{1}{6}\times 20\times 21\times 22 = 1540.00$

**25.** An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ....

**NTA Ans. (490.00)**

**Sol.** The question does not mention that whether same coloured marbles are distinct or identical. So, assuming they are distinct our required answer = ${}^{12}C_4 - {}^{5}C_4 = 490$

And, if same coloured marbles are identical then required answer = (2 + 3 + 4 + 4) = 13